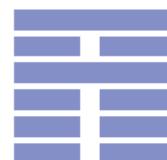


Optimal & Real Time Scheduling

— *using* —
Model Checking Technology

Kim G. Larsen



BRICS
Basic Research
in Computer Science



AIPS – a partial observation





April 2002 – June 2005 IST-2001-35304

■ Academic partners: ■ Industrial Partners

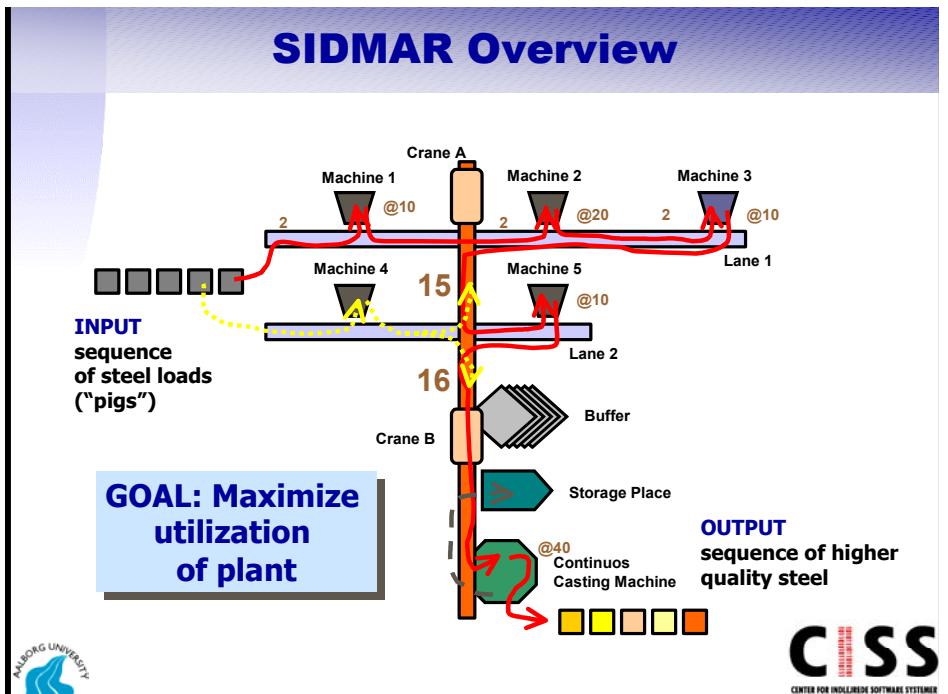
- Nijmegen
- Aalborg
- Dortmund
- Grenoble
- Marseille
- Twente
- Weizmann
- Axxom
- Bosch
- Cybernetix
- Terma

OBJECTIVES

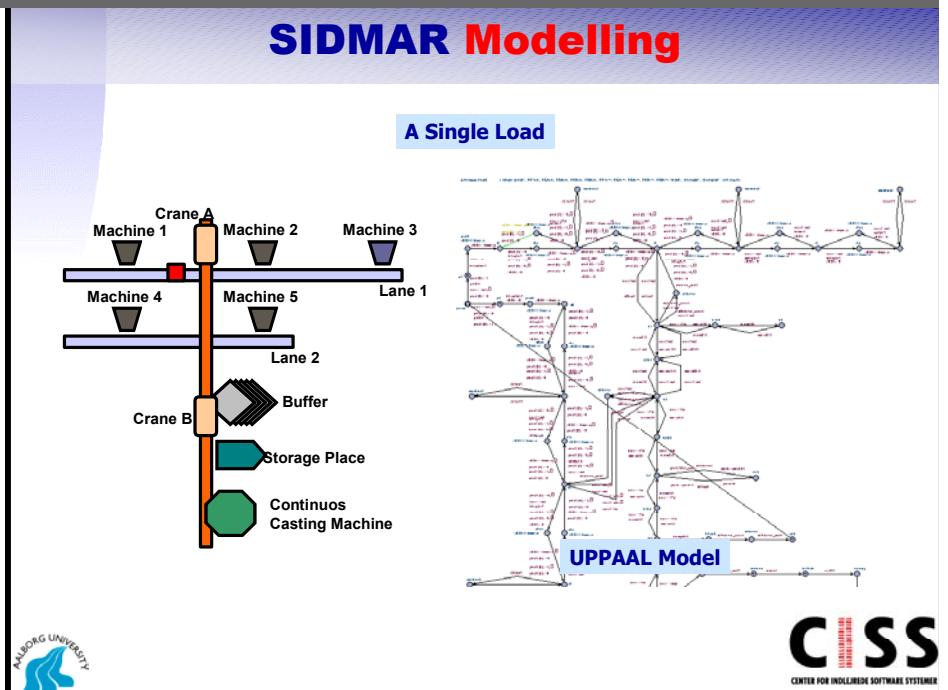
- powerful, unifying mathematical modelling
- efficient computerized problem-solving tools

- distributed real-time systems
- time-dependent behaviour and dynamic resource allocation

- **TIMED AUTOMATA**



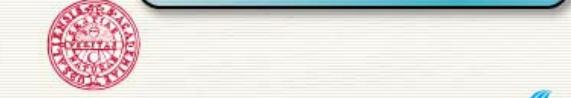
LTR Project VHS (Verification of Hybrid systems)



Case Studies

- Cybernetics
 - Smart Personal Assistant
- Terma:
 - Memory
- Bosch:
 - Car Park Sensors
- AXXOM
 - Lacquer

Benchmarks



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More information at <http://www.uppaal.com>.

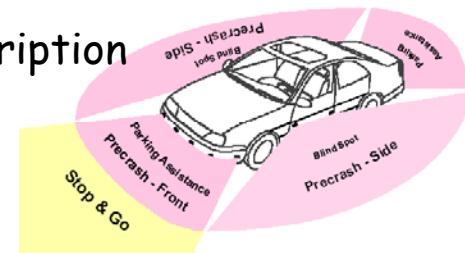
UPPAAL 3.4.7, Aug 2004.

CLASSIC

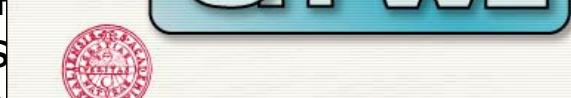
CPS: Informal description

- CPS obtains and makes available for other systems information about environment of a car. This information may be used for:

- Parking assistance
- Pre-crash detection
- Blind spot supervision
- Lane change assistance
- Stop & go
- Etc



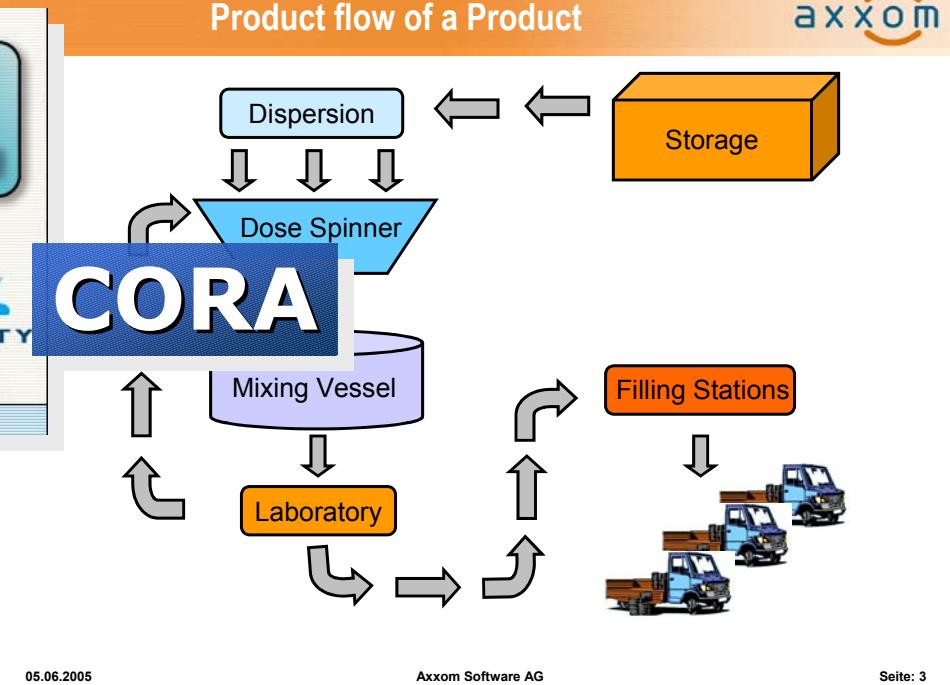
- The CPS considered in this case study
 - One sensor group only (currently 2 sensors)
 - Only the front sensors and corresponding controllers
 - Application: pre-crash detection, parking assistance, stop & go



Copyright 1995-2003 by Uppsala University and Aalborg University. All rights reserved.
More information at <http://www.uppaal.com>.

UPPAAL CORA 041109, Nov 2004.

F. Brinksmo Car Assembly Supervision System: Case Study 3 2



Overview

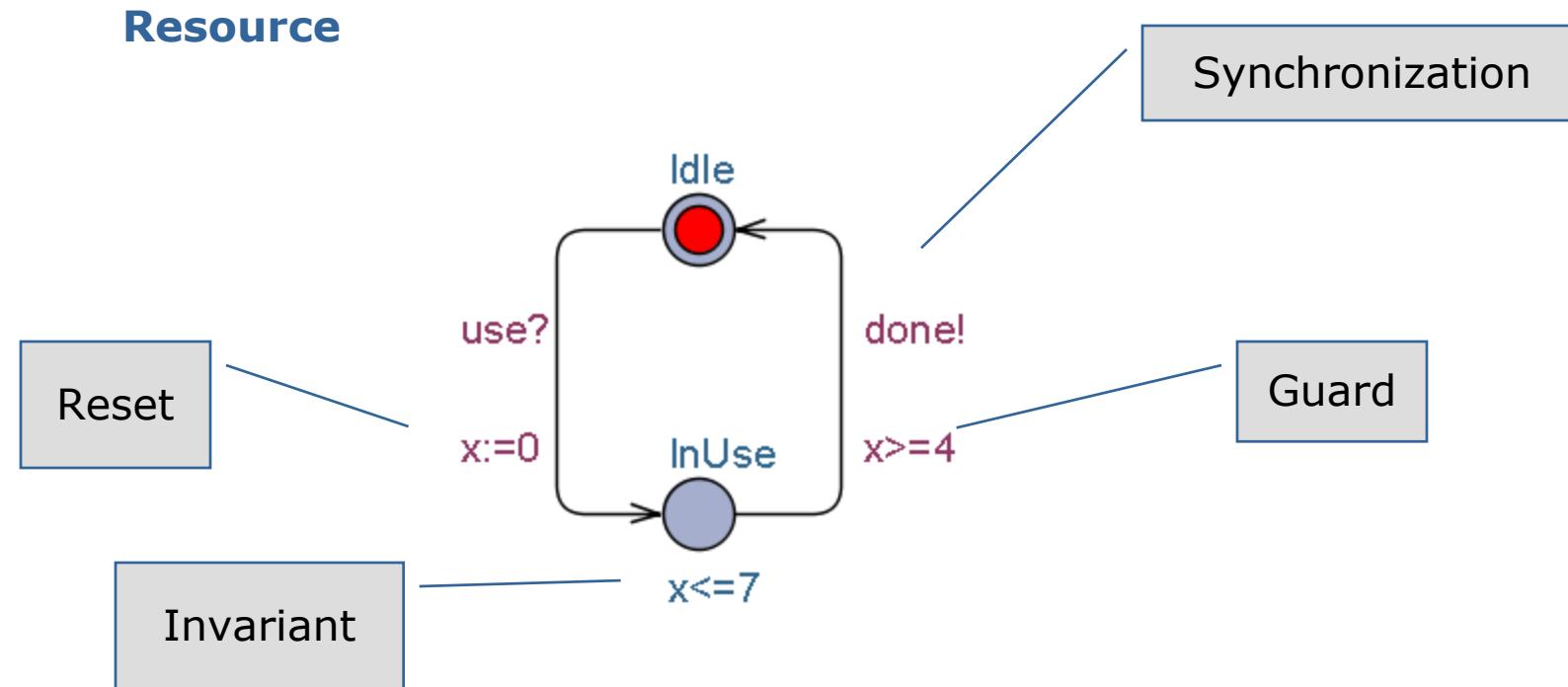
- Timed Automata & Scheduling
- Priced Timed Automata and Optimal Scheduling
- AMETIST Results
- Beyond Static Scheduling

Overview

- Timed Automata & Scheduling
- Priced Timed Automata and Optimal Scheduling
- AMETIST Results
- Beyond Static Scheduling

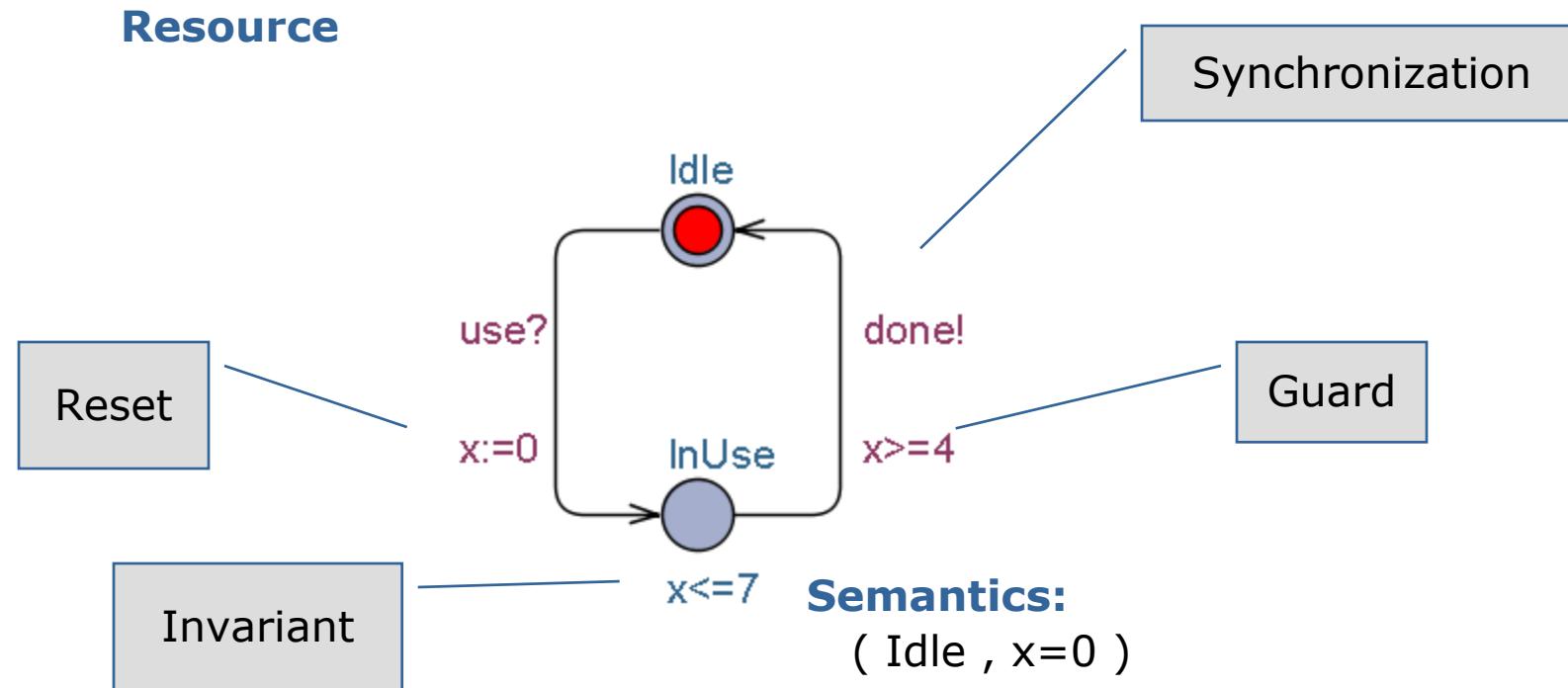
Timed Automata

[Alur & Dill'89]



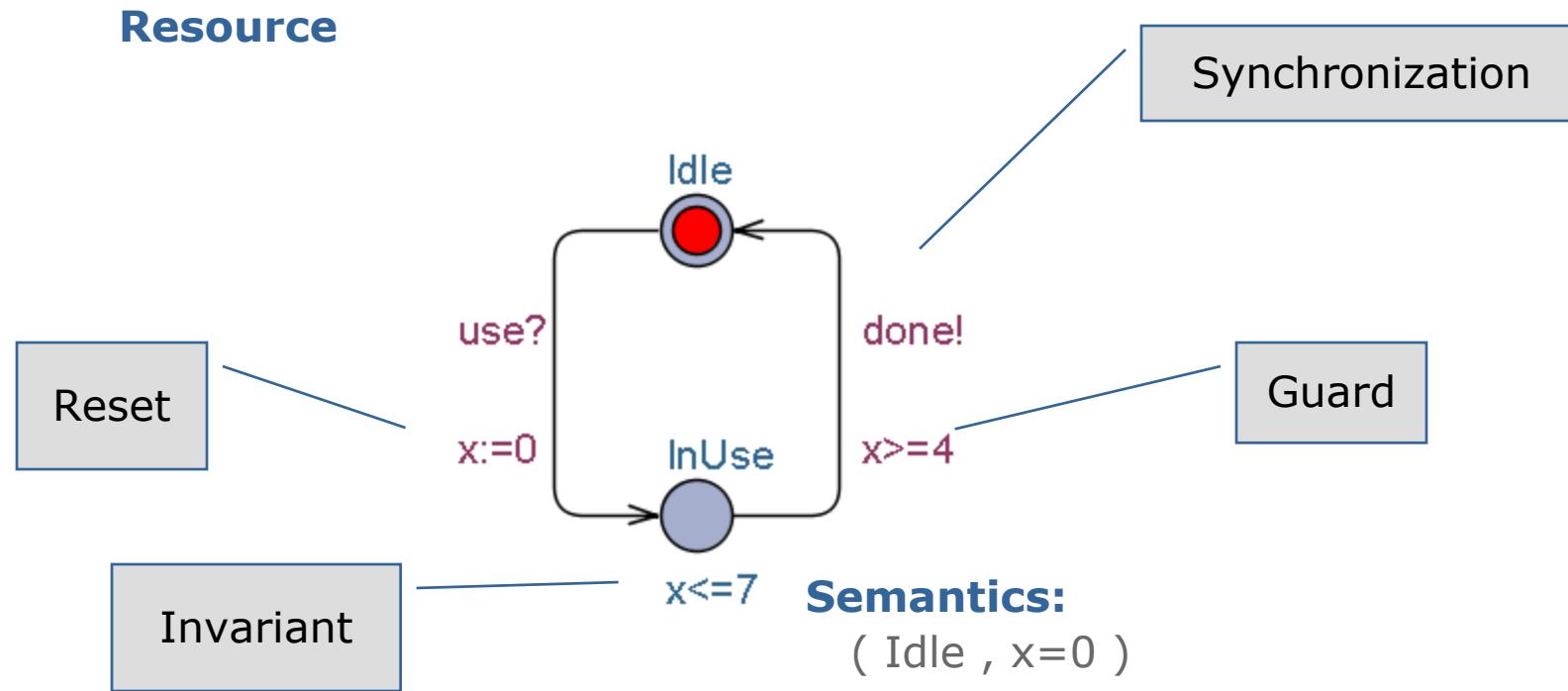
Timed Automata

[Alur & Dill'89]



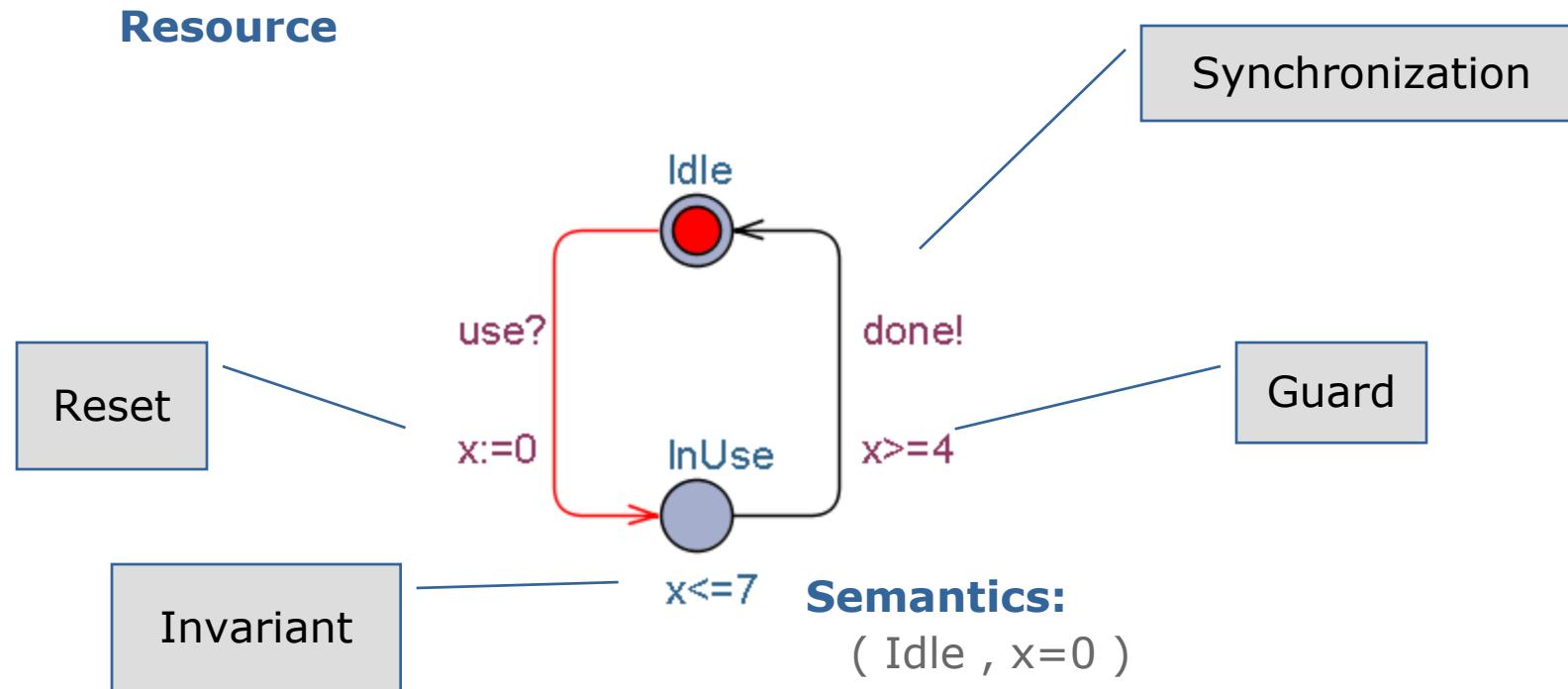
Timed Automata

[Alur & Dill'89]



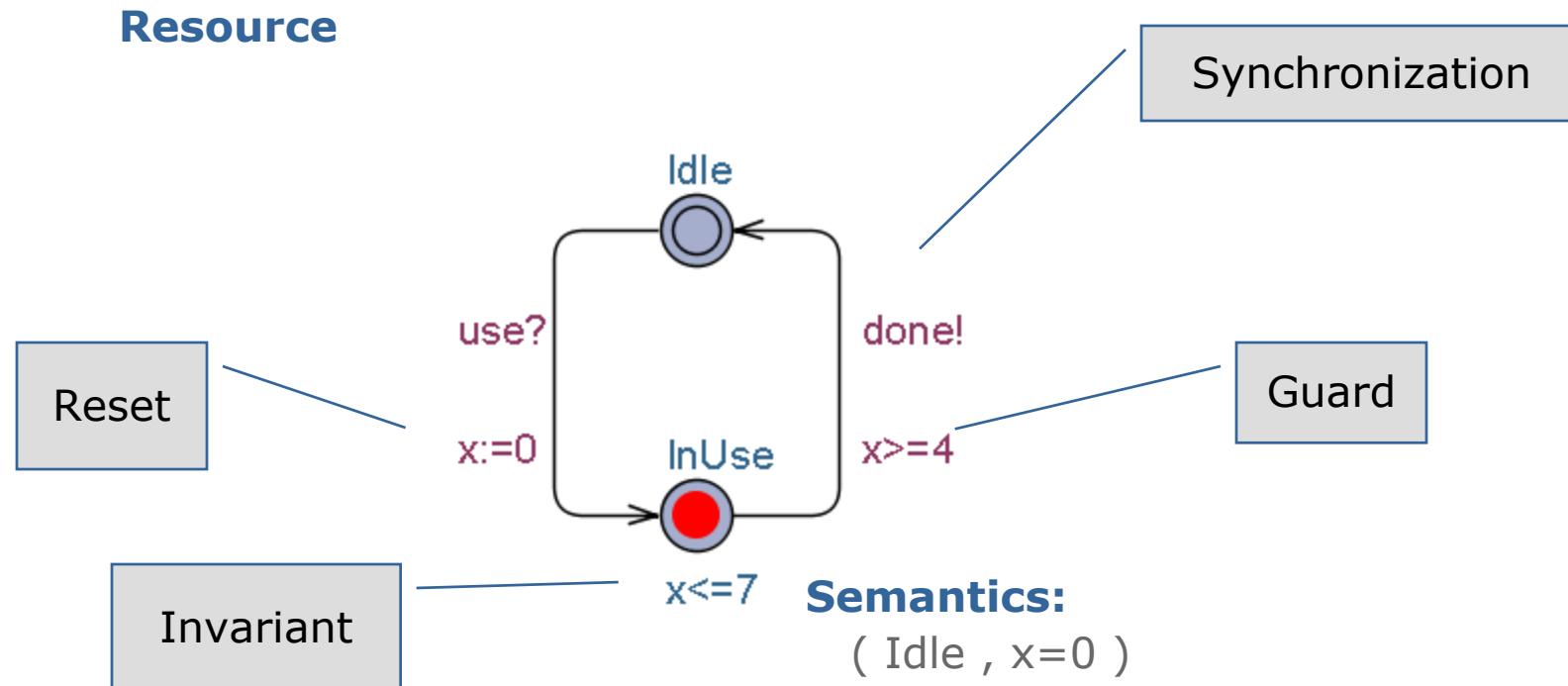
Timed Automata

[Alur & Dill'89]



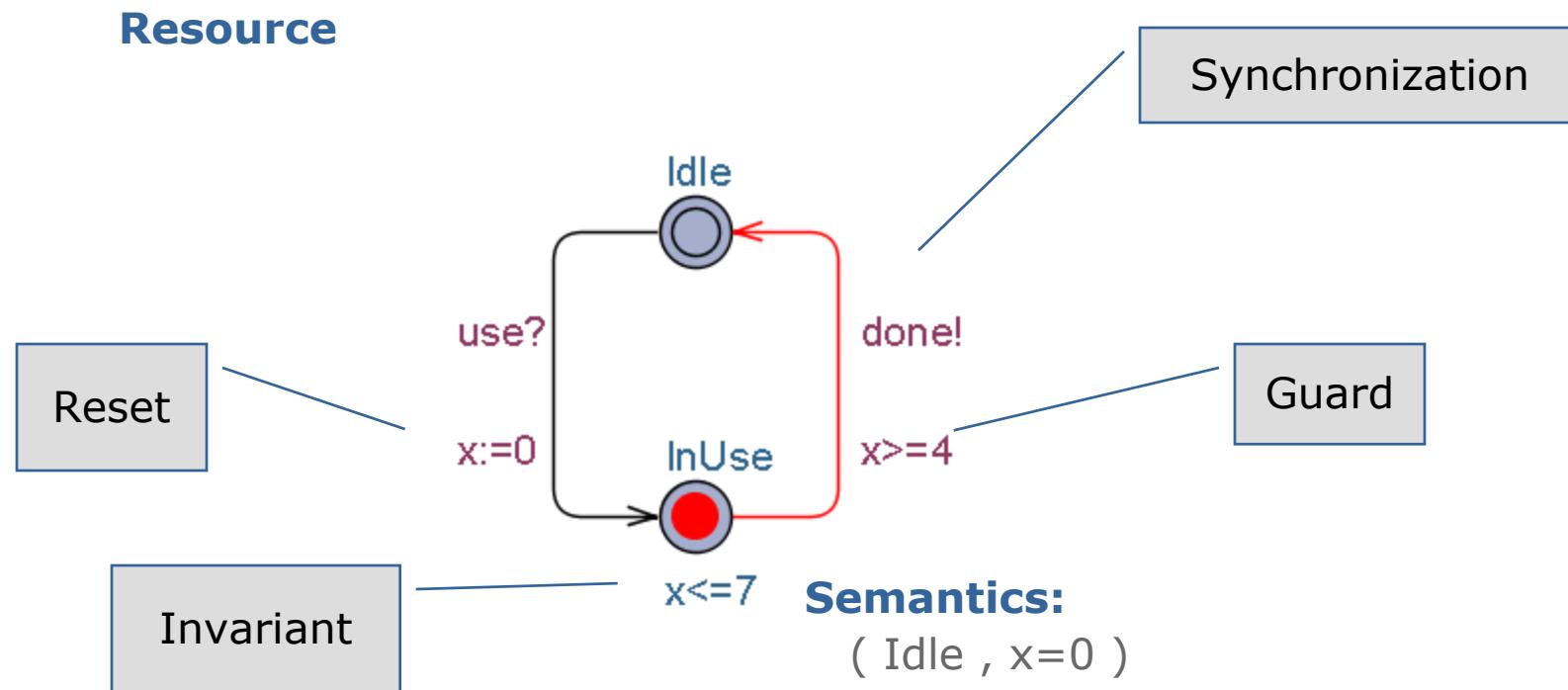
Timed Automata

[Alur & Dill'89]



Timed Automata

[Alur & Dill'89]

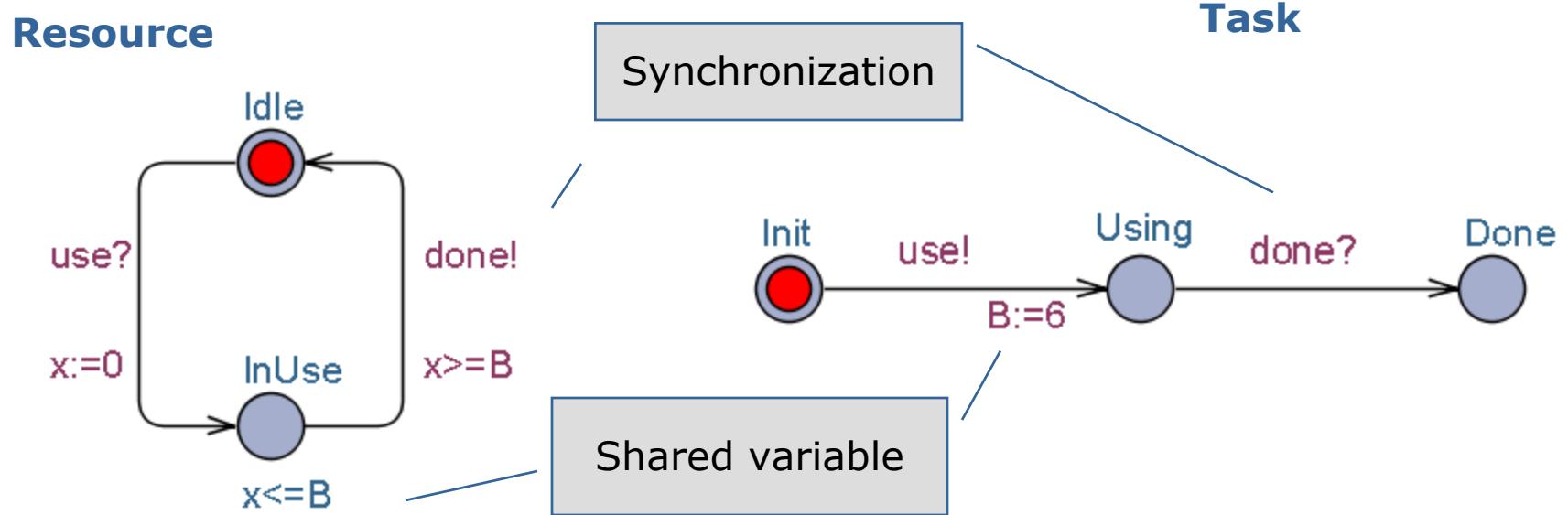


Semantics:

(Idle , $x=0$)

- (Idle , $x=2.5$) d(2.5)
- (InUse , $x=0$) use?
- (InUse , $x=5$) d(5)
- (Idle , $x=5$) done!
- (Idle , $x=8$) d(3)
- (InUse , $x=0$) use?

Composition

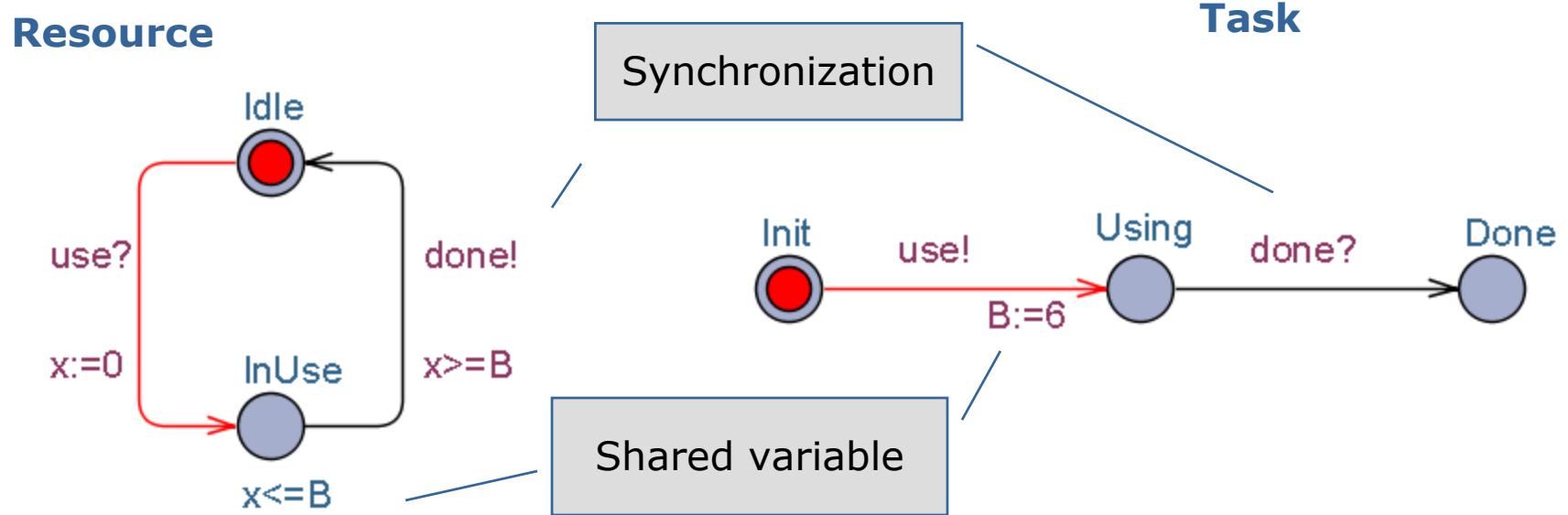


Semantics:

$(\text{Idle} , \text{Init} , B=0 , x=0)$
 $\rightarrow (\text{Idle} , \text{Init} , B=0 , x=3.1415) \quad d(3)$
 $\rightarrow (\text{InUse} , \text{Using} , B=6 , x=0)$
 $\rightarrow (\text{InUse} , \text{Using} , B=6 , x=6)$
 $\rightarrow (\text{Idle} , \text{Done} , B=6 , x=6)$

use
 $d(6)$
 done

Composition



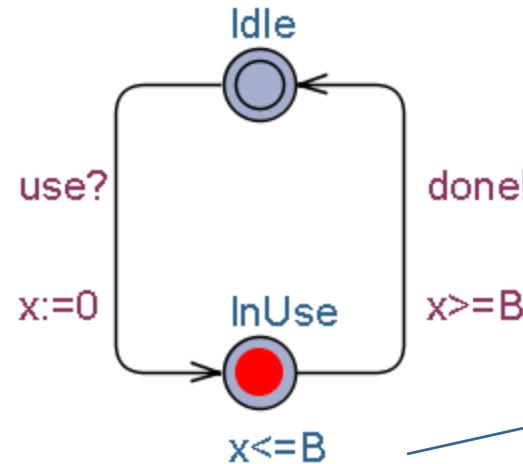
Semantics:

$(\text{Idle} , \text{Init} , B=0 , x=0)$
 $\rightarrow (\text{Idle} , \text{Init} , B=0 , x=3.1415) \quad d(3)$
 $\rightarrow (\text{InUse} , \text{Using} , B=6 , x=0)$
 $\rightarrow (\text{InUse} , \text{Using} , B=6 , x=6)$
 $\rightarrow (\text{Idle} , \text{Done} , B=6 , x=6)$

use
d(6)
done

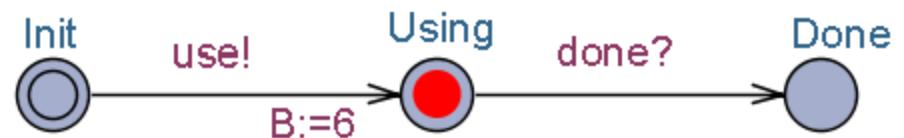
Composition

Resource



Synchronization

Task



Shared variable

Semantics:

(Idle , Init , B=0, x=0)

→ (Idle , Init , B=0 , x=3.1415) d(3)

→ (InUse , Using , B=6, x=0)

→ (InUse , Using , B=6, x=6)

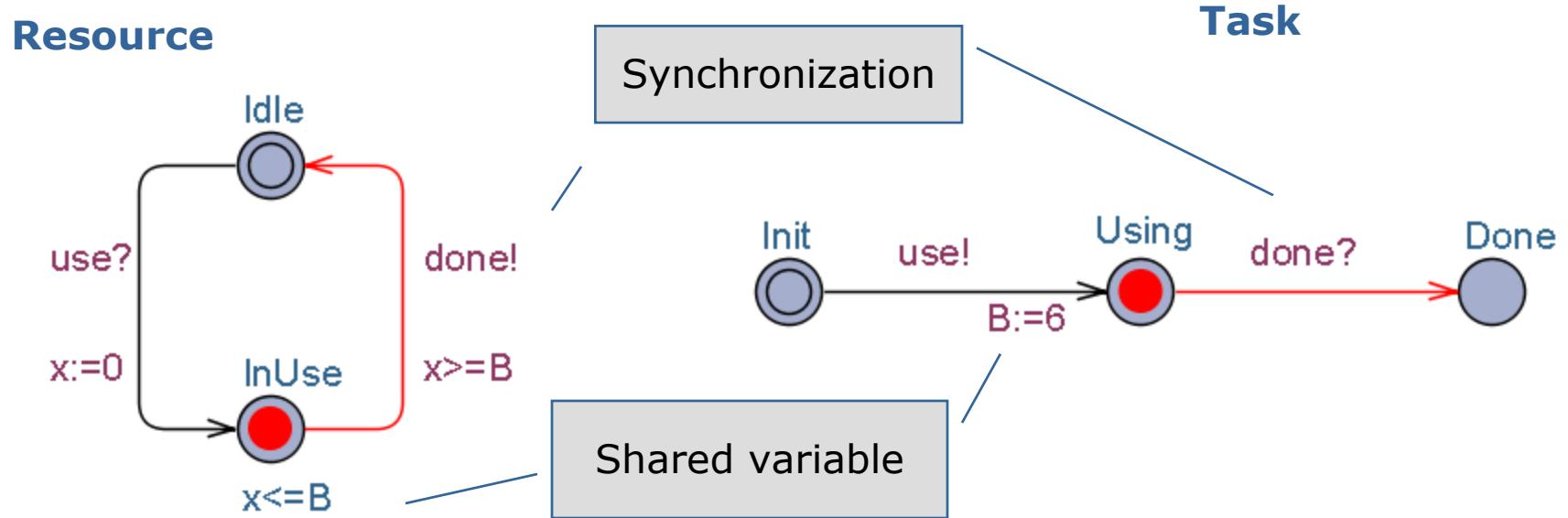
→ (Idle , Done , B=6 , x=6)

use

d(6)

done

Composition



Semantics:

(Idle , Init , B=0, x=0)

→ (Idle , Init , B=0 , x=3.1415) d(3)

→ (InUse , Using , B=6, x=0)

use

→ (InUse , Using , B=6, x=6)

d(6)

→ (Idle , Done , B=6 , x=6)

done

Jobshop Scheduling

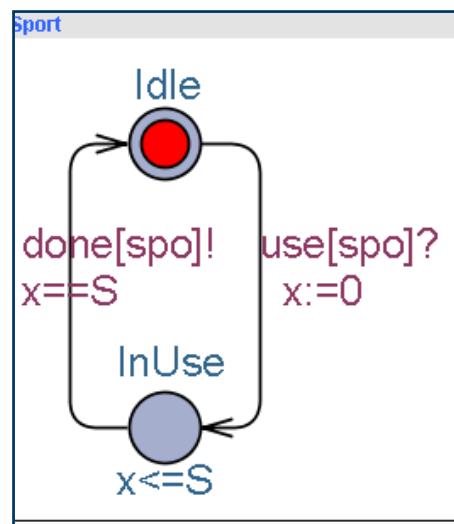
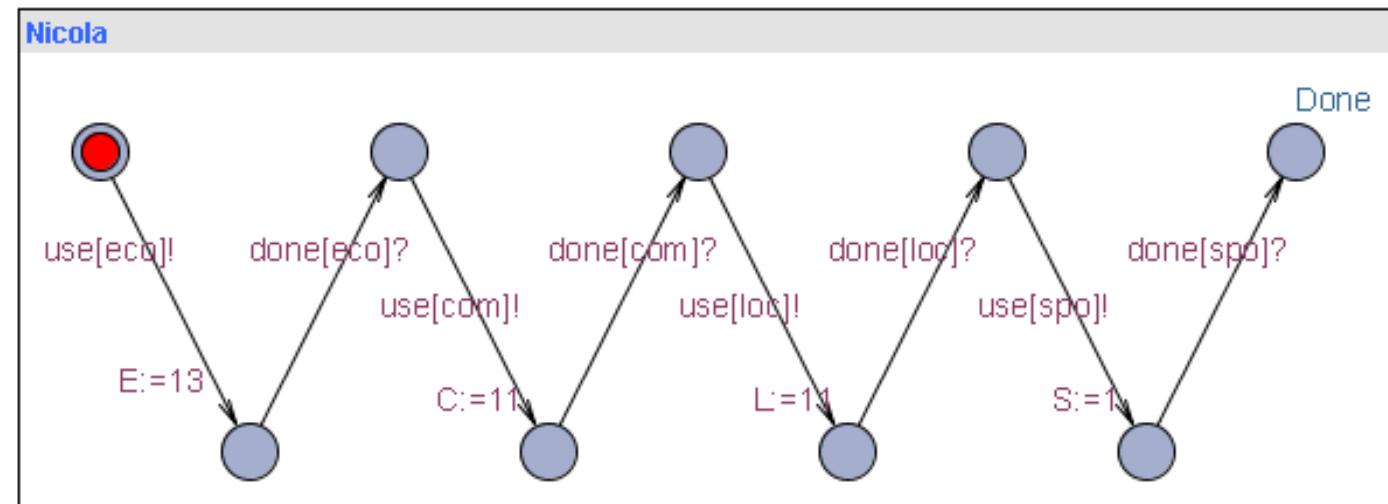
R E S O U R C E S

J O B S

	Sport	Economy	Local News	Comic Stip
Kim	2. 5 min	4. 1 min	3. 3 min	1. 10 min
Maria	1. 10 min	2. 20 min	3. 1 min	4. 1 min
Nicola	4. 1 min	1. 13 min	3. 11 min	2. 11 min

Problem: compute the minimal **MAKESPAN**

Jobshop Scheduling in UPPAAL



	Sport	Economy	Local News	Comic Strip
Kim	2. 5 min	4. 1 min	3. 3 min	1. 10 min
Maria	1. 10 min	2. 20 min	3. 1 min	4. 1 min
Nicola	4. 1 min	1. 13 min	3. 11 min	2. 11 min

Experiments

[TACAS'2001]

 B-&-B algorithm running
for 60 sec.

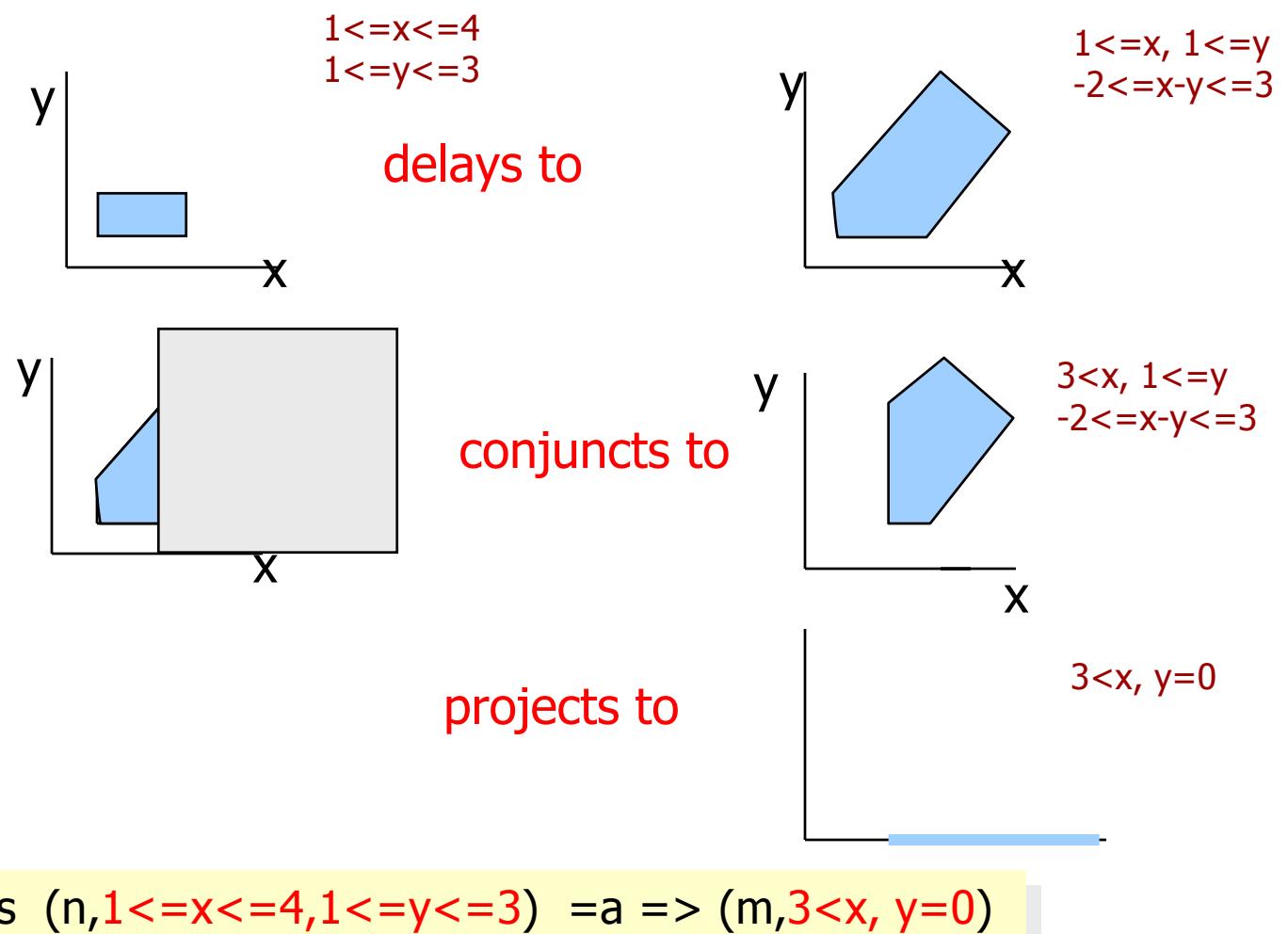
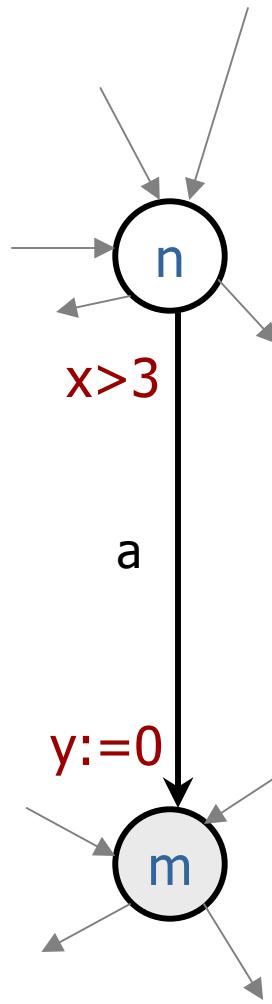
 m=5 j=10
 m=10 j=10
 m=15 j=10
 m=5 j=15
 m=10 j=20
 m=15 j=20
 m=5 j=20

problem instance	BF		MC		MC+		DF		RDF		comb. heur.		minimal makespan
	cost	states	cost	states									
la01	-	-	-	-	-	-	2466	-	842	-	666	292	666
la02	-	-	-	-	-	-	2360	-	806	-	672	-	655
la03	-	-	-	-	-	-	2094	-	769	-	626	-	597
la04	-	-	-	-	-	-	2212	-	783	-	639	-	590
la05	-	-	-	-	593	9791	1955	-	696	-	593	284	593
la06	-	-	-	-	-	-	3656	-	1076	-	926	480	926
la07	-	-	-	-	-	-	3410	-	1113	-	890	-	890
la08	-	-	-	-	-	-	3520	-	1009	-	863	400	863
la09	-	-	-	-	-	-	3984	-	1154	-	951	425	951
la10	-	-	-	-	-	-	3681	-	1063	-	958	454	958
la11	-	-	-	-	-	-	4974	-	1303	-	1222	642	1222
la12	-	-	-	-	-	-	4557	-	1271	-	1039	633	1039
la13	-	-	-	-	-	-	4846	-	1227	-	1150	662	1150
la14	-	-	-	-	1292	10653	5145	-	1377	-	1292	688	1292
la15	-	-	-	-	-	-	5264	-	1459	-	1289	-	1207
la16	-	-	-	-	-	-	4849	-	1298	-	1022	-	945
la17	-	-	-	-	-	-	4299	-	938	-	786	-	784
la18	-	-	-	-	-	-	4763	-	1034	-	922	-	848
la19	-	-	-	-	-	-	4566	-	1140	-	904	-	842
la20	-	-	-	-	-	-	5056	-	1378	-	964	-	902
la21	-	-	-	-	-	-	7608	-	1326	-	1149	-	(1040,1053)
la22	-	-	-	-	-	-	6920	-	1413	-	1047	-	927
la23	-	-	-	-	-	-	7676	-	1357	-	1075	-	1032
la24	-	-	-	-	-	-	7237	-	1346	-	1061	-	935
la25	-	-	-	-	-	-	7141	-	1290	-	1070	-	977

Lawrence Job Shop Problems

Symbolic Transitions

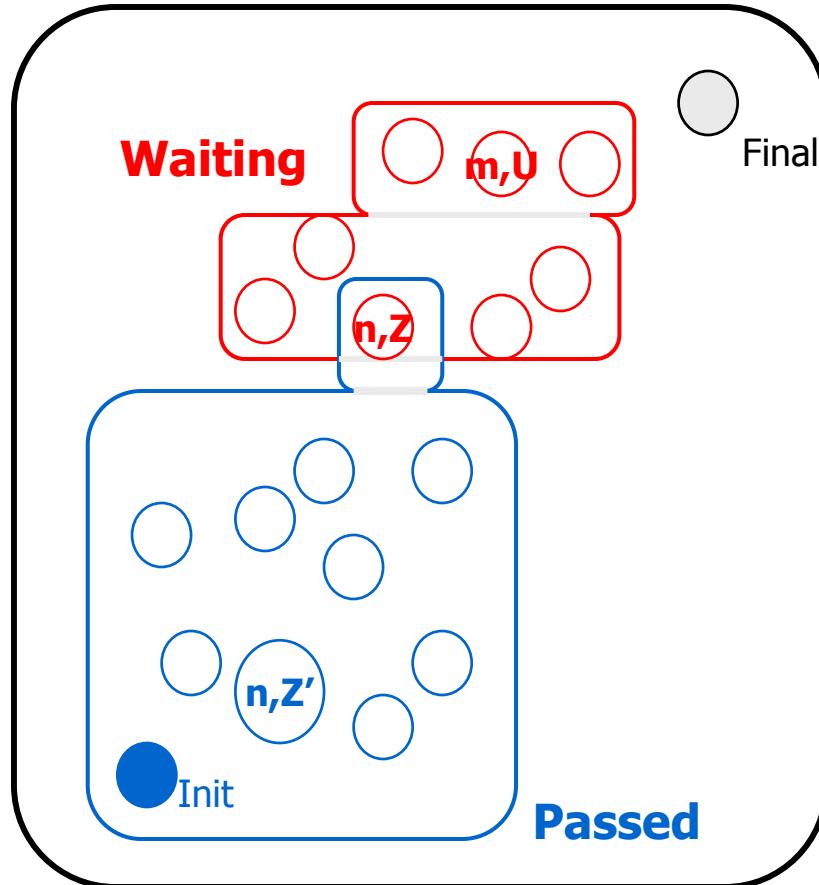
using Zones



Thus $(n, 1 \leq x \leq 4, 1 \leq y \leq 3) = a \Rightarrow (m, 3 < x, y = 0)$

Forward Rechability

Init \rightarrow Final ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT

- pick (n, Z) in **Waiting**
- if for some $Z' \supseteq Z$
 (n, Z') in **Passed** then **STOP**
- else /explore/ add
 $\{ (m, U) : (n, Z) \Rightarrow (m, U) \}$
to **Waiting**;
- Add (n, Z) to **Passed**

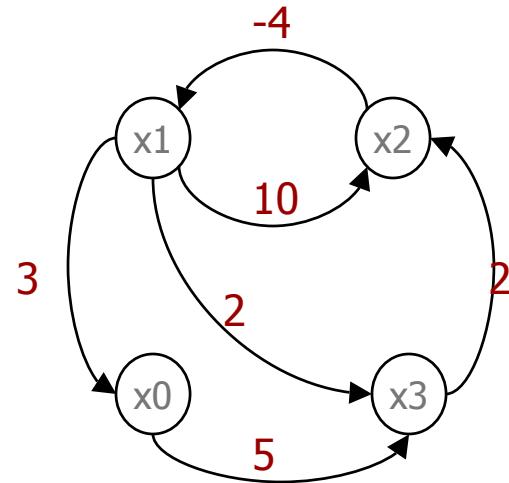
UNTIL **Waiting** = \emptyset

or

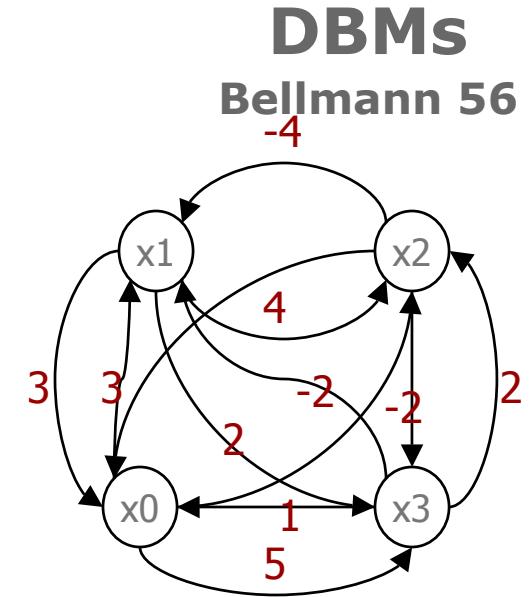
Final is in **Waiting**

Canonical Datastructures for Zones

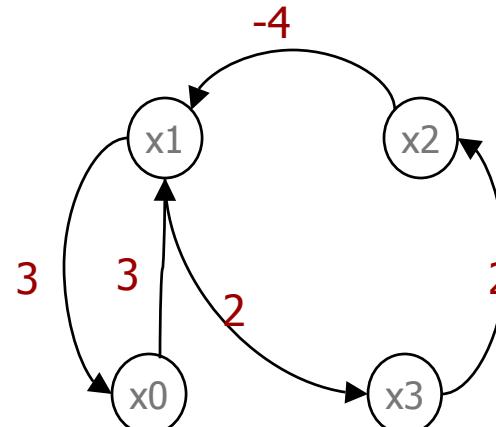
```
x1-x2<=4
x2-x1<=10
x3-x1<=2
x2-x3<=2
x0-x1<=3
x3-x0<=5
```



Shortest Path Closure
 $O(n^3)$



Shortest Path Reduction
 $O(n^3)$



Space worst $O(n^2)$
practice $O(n)$

Minimal Constraint Form
RTSS'97

Datastructures for Zones

■ DRM package

UPPAAL - Mozilla

File Edit View Go Bookmarks Tools Window Help

Home Bookmarks

 **DBM Library**

For more information about DBMs see the [DBM Library](#).

Current version is **1.0.2**.

 Download for Linux

Documentation

-  API of the library
- Related presentation on the library and subtractions:
-  Presentation (ppt)
-  Presentation (pdf)
- Related papers:

NEWS

04/02/2004: Version 1.0.2 released. Changes are: new functions for federations and bug fixes for Federation, dbmf_predit, and the subtractions (rare bug).
 22/12/2004: UPPAAL DBM library 1.0.1 released.

UPPAAL DBM LIBRARY

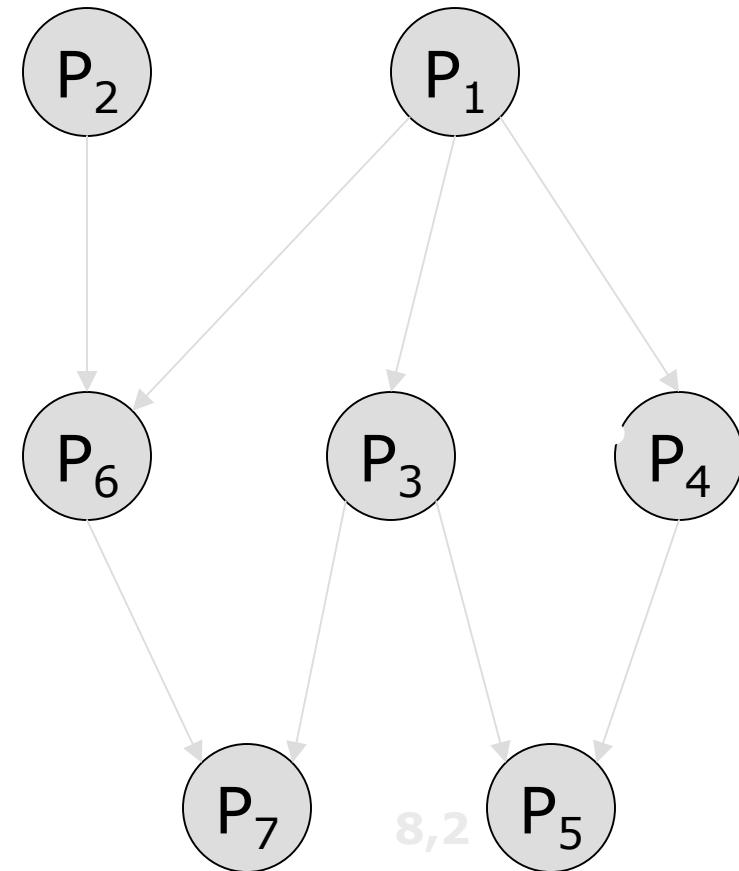
DBMs (difference bound matrices) [rokicki93, lpw:fct95, bengtsson02] are efficient data structures to represent clock constraints in timed automata [ad90]. They are used in UPPAAL [ipy97, b04, bdl04] as the core data structure to represent time. The library features all the common operations such as up (delay, or future), down (past), general

X True

Task Graph Scheduling

Optimal Static Task Scheduling

-
-
-
-
-
-
-
-
-
-

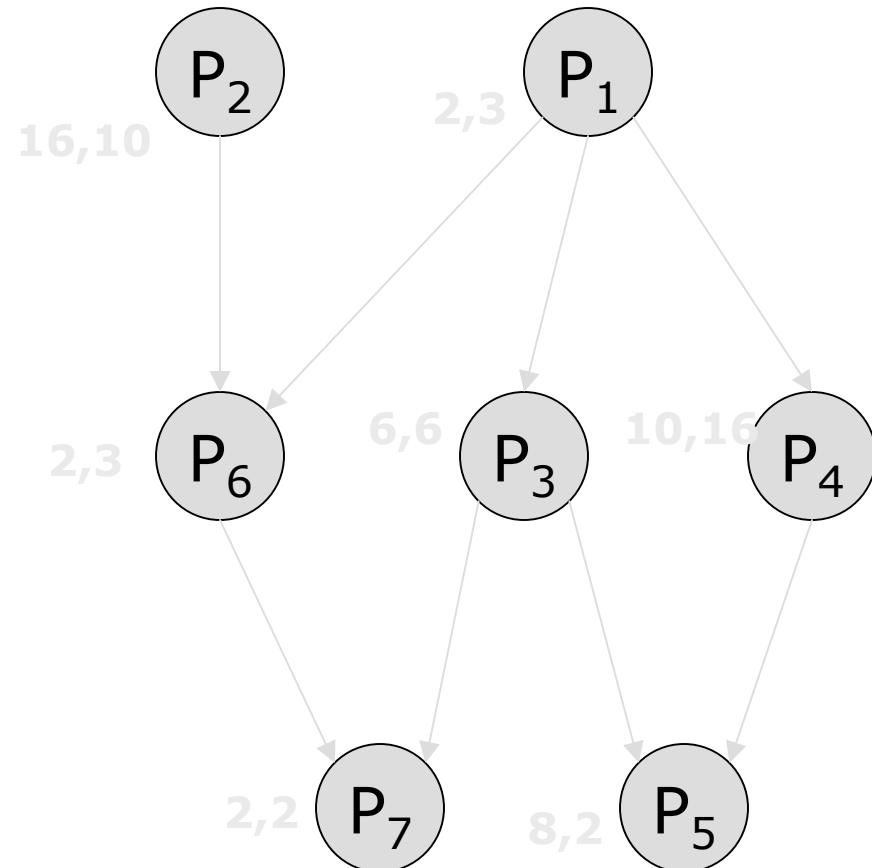


$$\mathbf{M} = \{\mathbf{M}_1, \mathbf{M}_2\}$$

Task Graph Scheduling

Optimal Static Task Scheduling

- Task $P = \{P_1, \dots, P_m\}$
- Machines $M = \{M_1, \dots, M_n\}$
-
-
-
-
-
-
-
-



$$M = \{M_1, M_2\}$$



Task Graph Scheduling

Optimal Static Task Scheduling

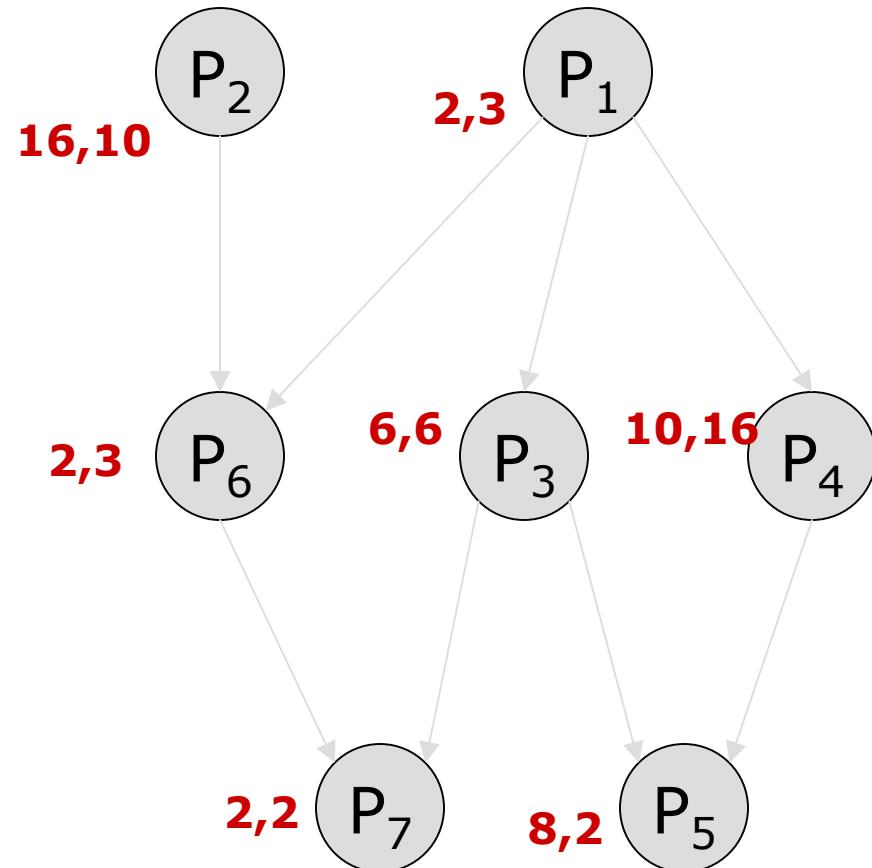
- Task $P = \{P_1, \dots, P_m\}$
- Machines $M = \{M_1, \dots, M_n\}$
- Duration $\Delta : (P \times M) \rightarrow \mathbb{N}_\infty$

■

■

■

■



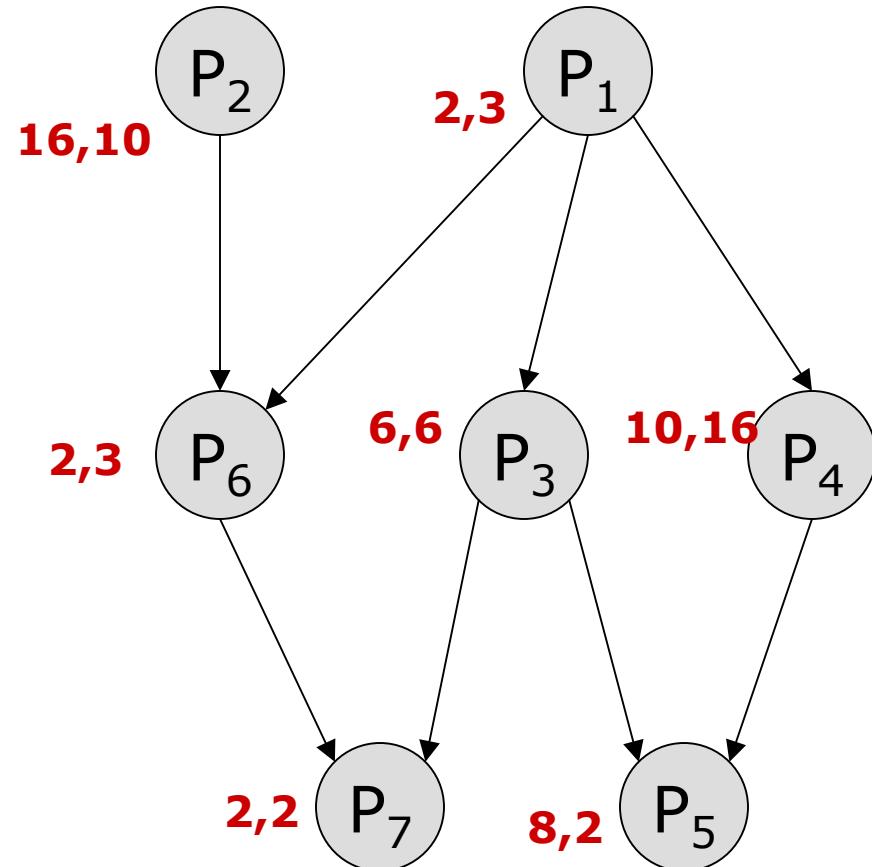
$$M = \{M_1, M_2\}$$



Task Graph Scheduling

Optimal Static Task Scheduling

- Task $\mathbf{P} = \{P_1, \dots, P_m\}$
- Machines $\mathbf{M} = \{M_1, \dots, M_n\}$
- Duration $\Delta : (\mathbf{P} \times \mathbf{M}) \rightarrow \mathbb{N}_\infty$
- $<$: p.o. on \mathbf{P} (pred.)



$\mathbf{M} = \{M_1, M_2\}$

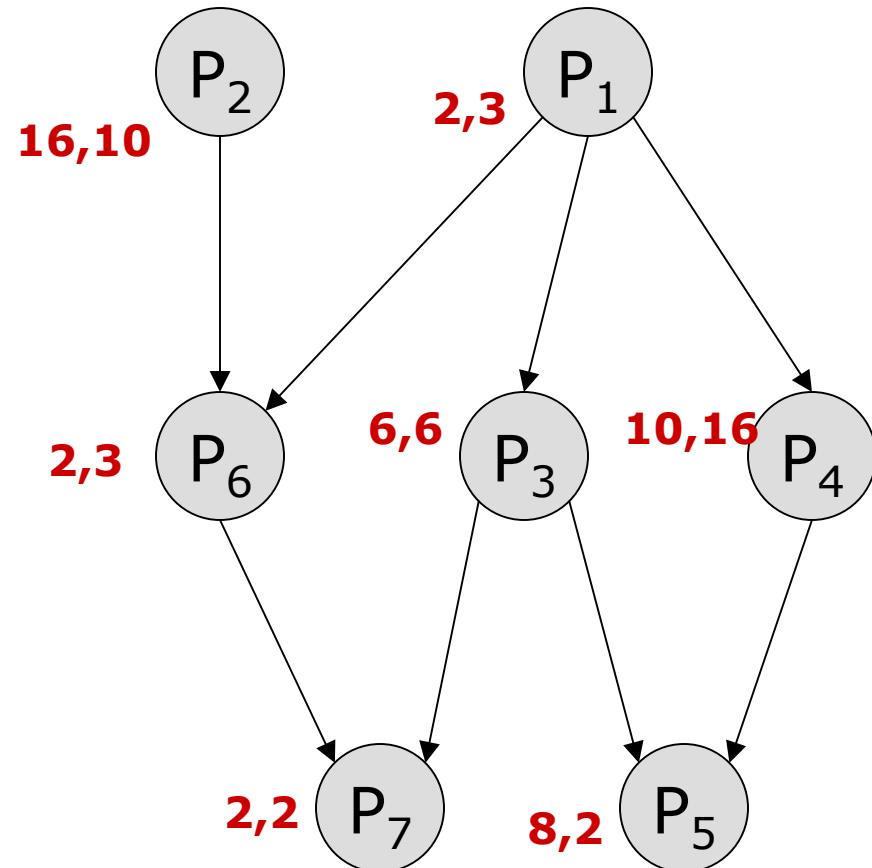


Task Graph Scheduling

Optimal Static Task Scheduling

- Task $\mathbf{P} = \{P_1, \dots, P_m\}$
- Machines $\mathbf{M} = \{M_1, \dots, M_n\}$
- Duration $\Delta : (\mathbf{P} \times \mathbf{M}) \rightarrow \mathbb{N}_\infty$
- $<$: p.o. on \mathbf{P} (pred.)

- A task can be executed only if all predecessors have completed
- Each machine can process at most one task at a time
- Task cannot be preempted.



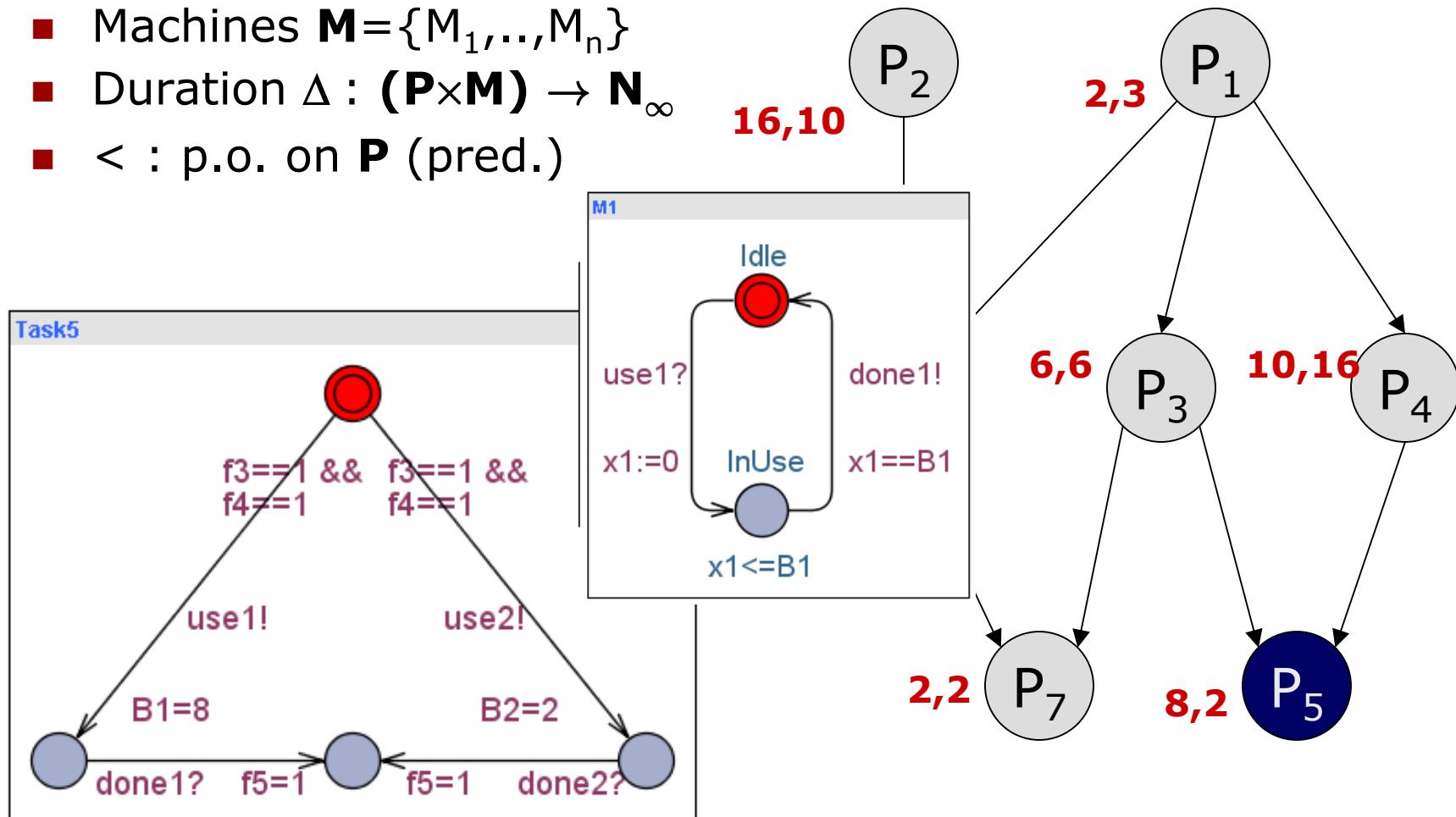
$$\mathbf{M} = \{M_1, M_2\}$$



Task Graph Scheduling

Optimal Static Task Scheduling

- Task $\mathbf{P} = \{P_1, \dots, P_m\}$
- Machines $\mathbf{M} = \{M_1, \dots, M_n\}$
- Duration $\Delta : (\mathbf{P} \times \mathbf{M}) \rightarrow \mathbb{N}_\infty$
- $<$: p.o. on \mathbf{P} (pred.)



$$\mathbf{M} = \{M_1, M_2\}$$

Experimental Results

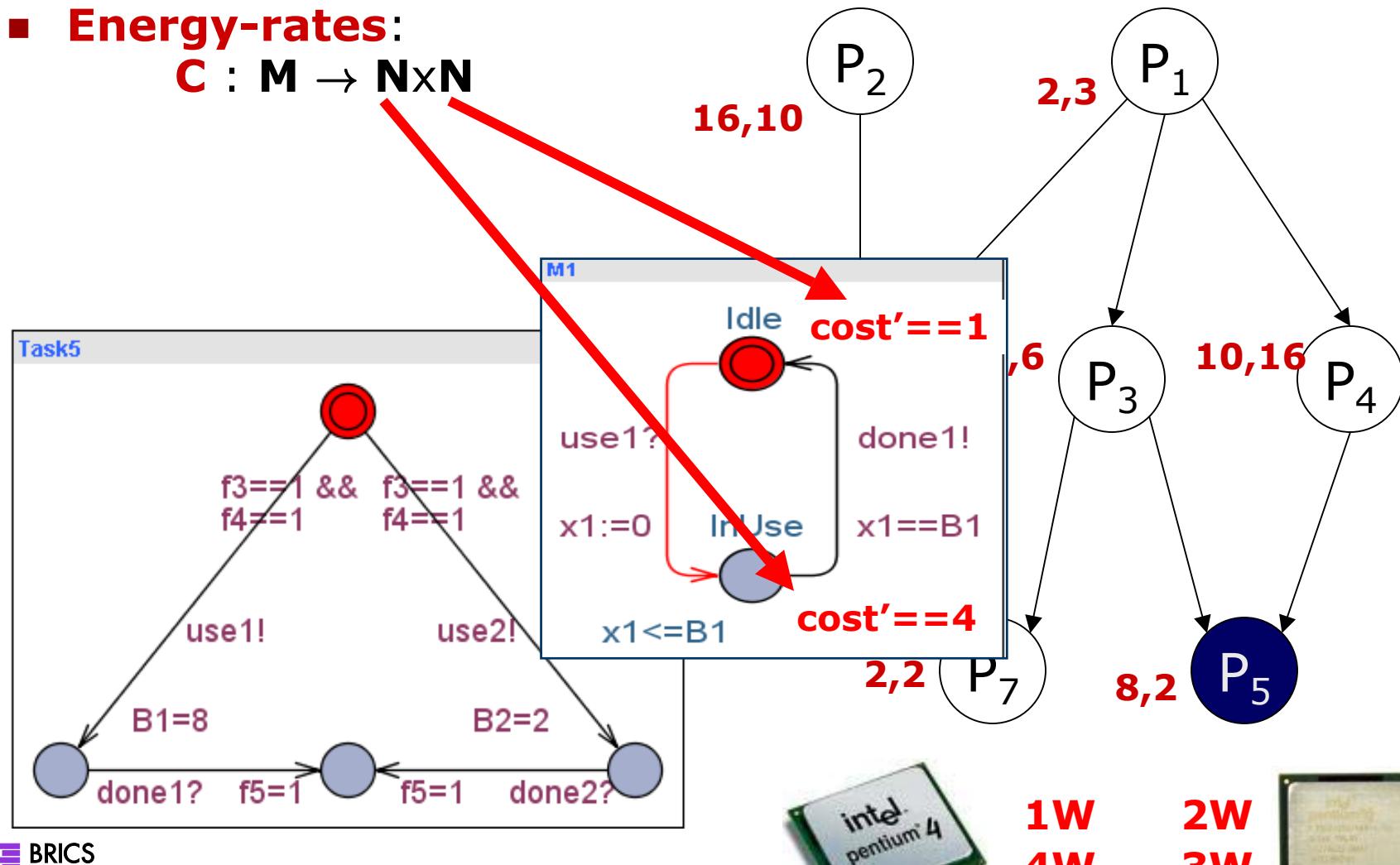
name	#tasks	#chains	# machines	optimal	TA
001	437	125	4	1178	1182
000	452	43	20	537	537
018	730	175	10	700	704
074	1007	66	12	891	894
021	1145	88	20	605	612
228	1187	293	8	1570	1574
071	1193	124	20	629	634
271	1348	127	12	1163	1164
237	1566	152	12	1340	1342
231	1664	101	16	t.o.	1137
235	1782	218	16	t.o.	1150
233	1980	207	19	1118	1121
294	2014	141	17	1257	1261
295	2168	965	18	1318	1322
292	2333	318	3	8009	8009
298	2399	303	10	2471	2473

Optimal Task Graph Scheduling

Power-Optimality

- **Energy-rates:**

$$C : M \rightarrow N \times N$$



1W
4W



2W
3W

Overview

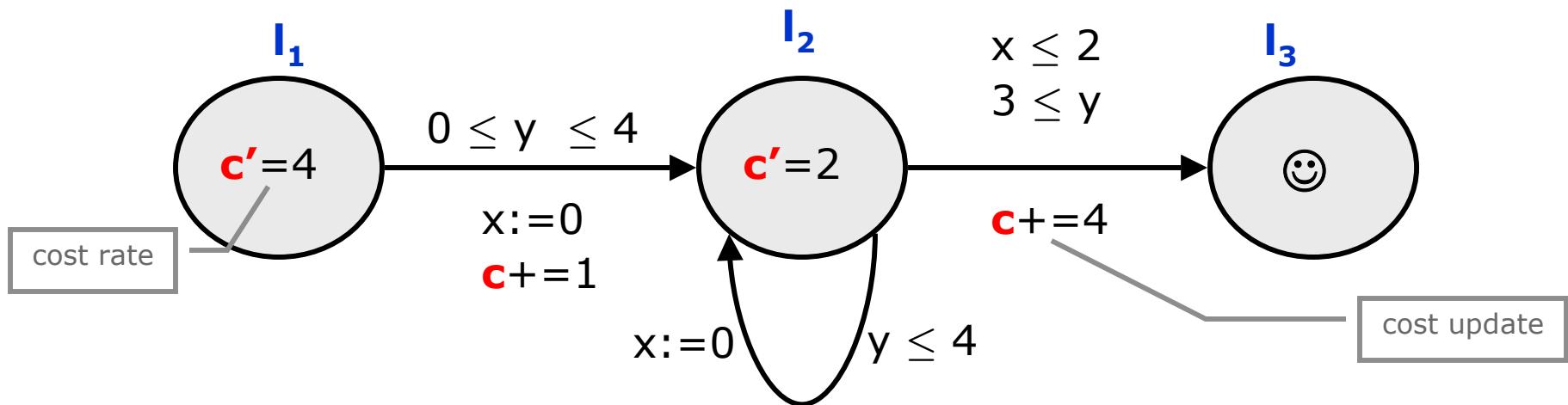
- Timed Automata & Scheduling
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Priced Timed Automata

Timed Automata + **COST** variable

Behrmann, Fehnker, et all (HSCC'01)

Alur, Torre, Pappas (HSCC'01)

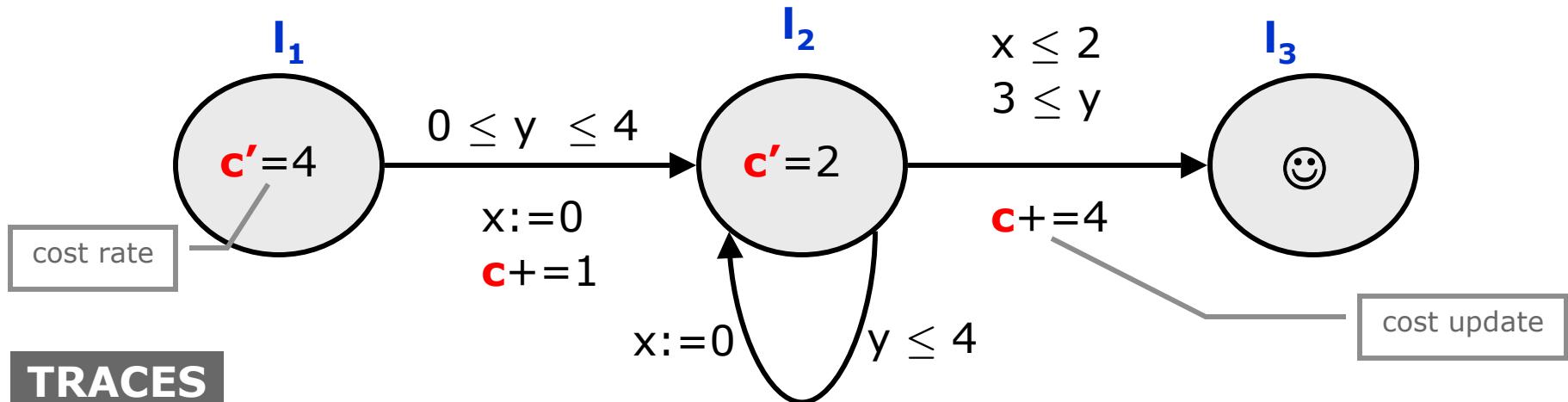


Priced Timed Automata

Timed Automata + **COST** variable

Behrmann, Fehnker, et all (HSCC'01)

Alur, Torre, Pappas (HSCC'01)



$$(\textcolor{blue}{I_1}, x=y=0) \xrightarrow[12]{\varepsilon(3)} (\textcolor{blue}{I_1}, x=y=3) \xrightarrow[1]{} (\textcolor{blue}{I_2}, x=0, y=3) \xrightarrow[4]{} (\textcolor{blue}{I}_3, _, _)$$

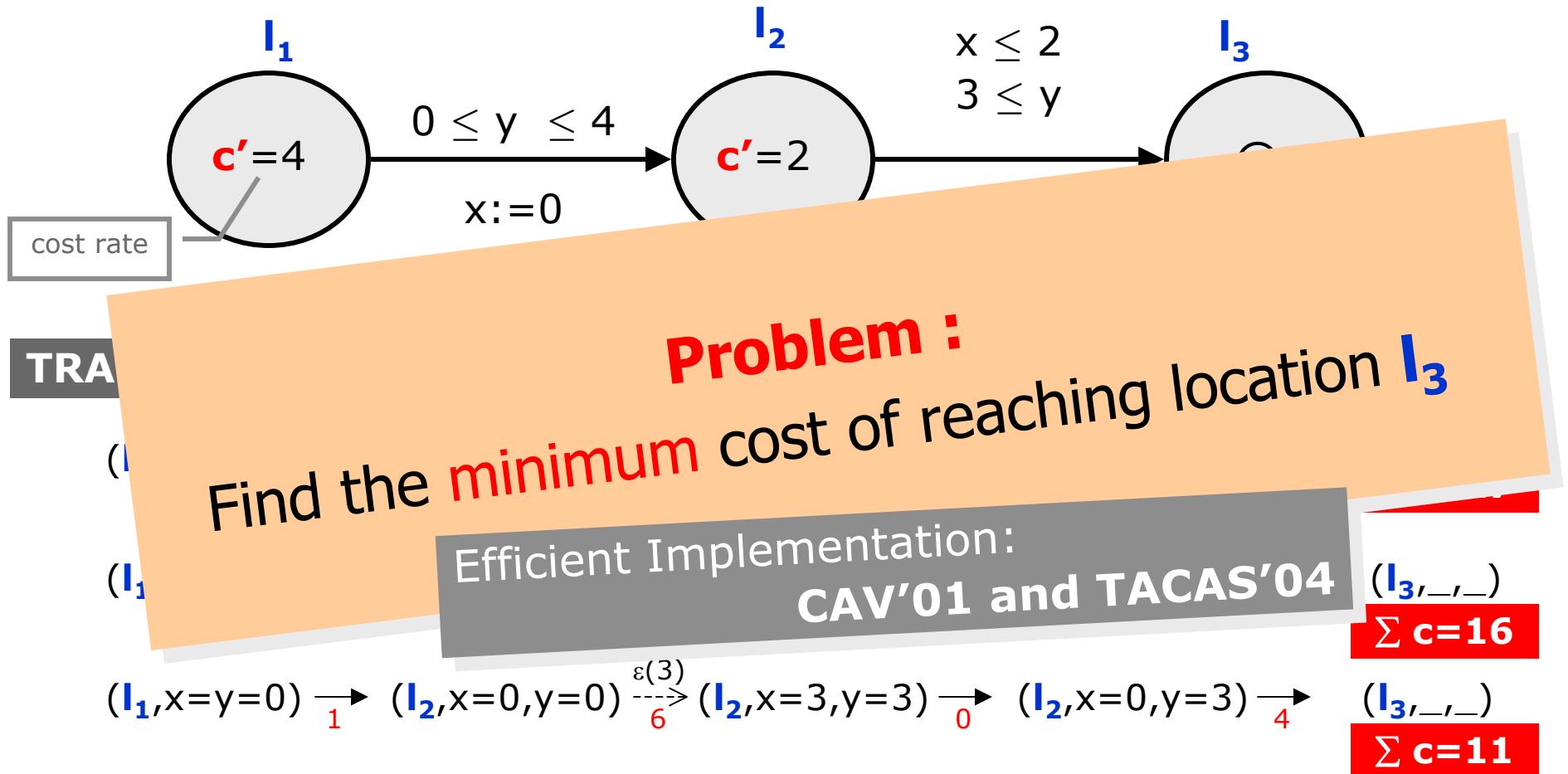
$\Sigma c=17$

Priced Timed Automata

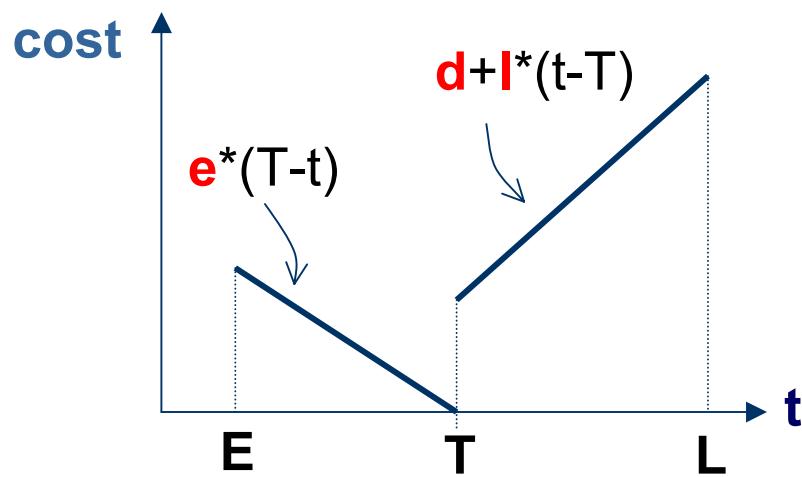
Timed Automata + **COST** variable

Behrmann, Fehnker, et all (HSCC'01)

Alur, Torre, Pappas (HSCC'01)



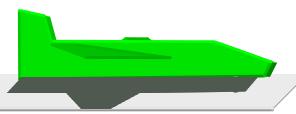
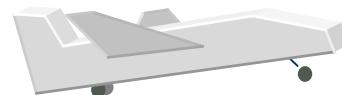
Aircraft Landing Problem



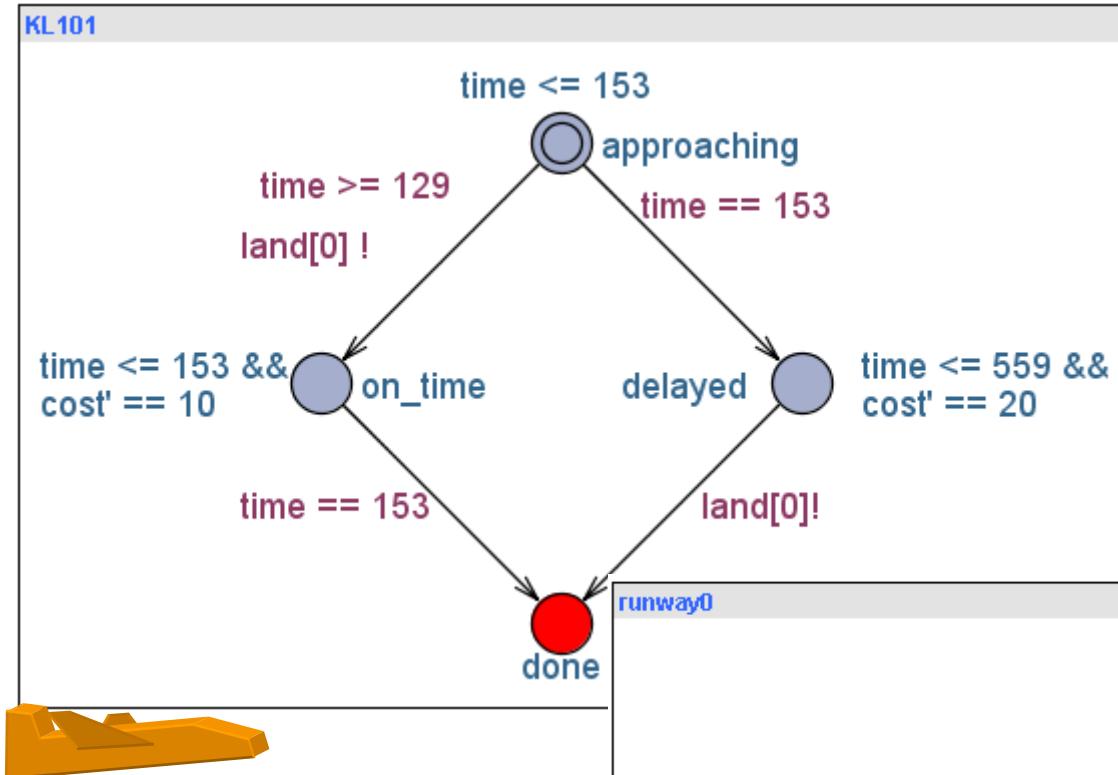
- E** earliest landing time
- T** target time
- L** latest time
- e** cost rate for being early
- I** cost rate for being late
- d** fixed cost for being late



Planes have to keep separation distance to avoid turbulences caused by preceding planes



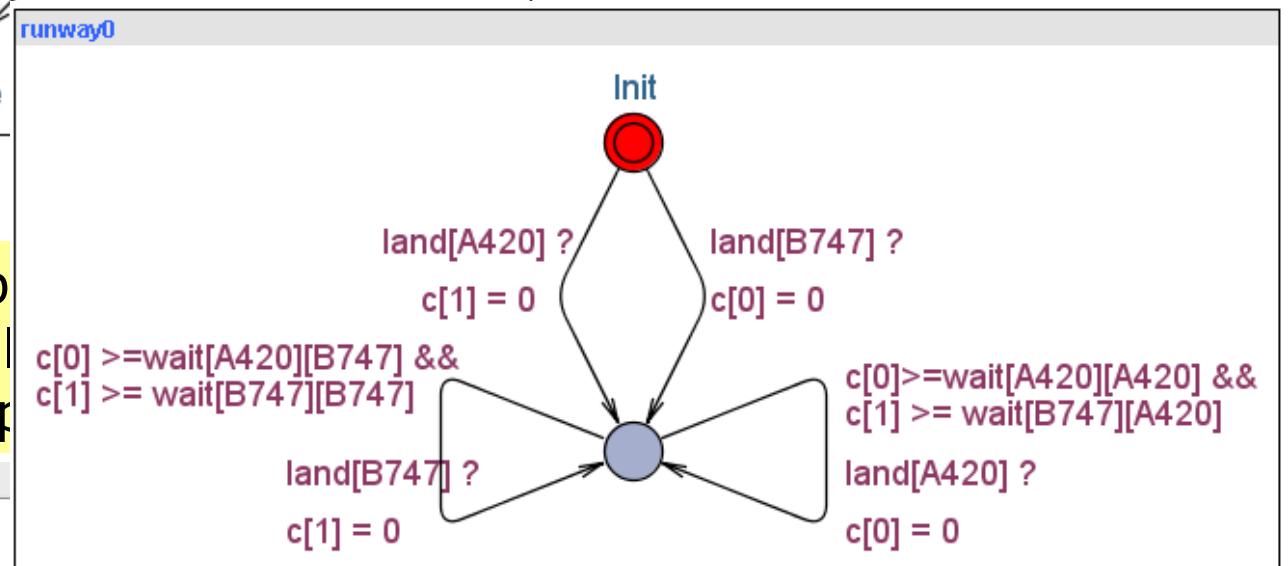
Modeling ALP with PTA



Planes have to keep sep distance to avoid turbulence caused by preceding plane

129: Earliest landing time
 153: Target landing time
 559: Latest landing time
 10: Cost rate for early
 20: Cost rate for late

Runway handles 2 types of planes



Symbolic "A"

Zones

Definition

A **zone** Z over a set of clocks C is a finite conjunction of simple constraints of the forms:

$$x \geq l \quad x \leq u \quad x - y \geq l' \quad x - y \leq u'$$

where $x, y \in C$, $l, u \in \mathbb{N}$ and $l', u' \in \mathbb{Z}$.

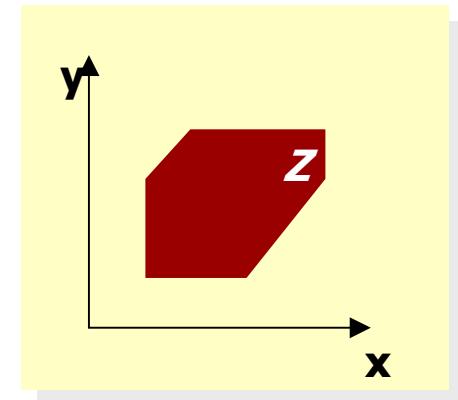
For $u \in \mathbb{R}^C$ and Z a zone we write $u \models Z$ if u satisfies all constraints of Z .

Operations

Reset: $\{x\}Z = \{u[0/x] \mid u \models Z\}$

Delay: $Z^\uparrow = \{u + d \mid u \models Z\}$

Offset: $\Delta_Z \models Z$ such that $\forall u \models Z. \forall x \in C. \Delta_Z(x) \leq u(x)$.



Priced Zone

CAV'01

Definition

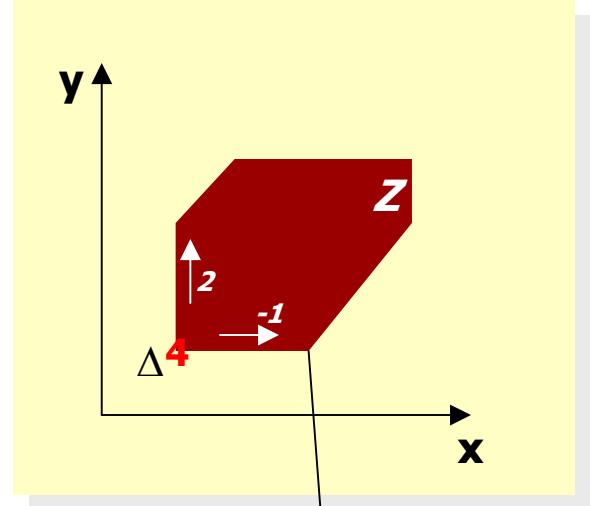
A priced zone P is a tuple (Z, c, r) , where:

- Z is a zone
- $c \in \mathbb{N}$ describes the cost of Δ_Z
- $r : C \rightarrow \mathbb{Z}$ gives a **rate** for any clock $x \in C$.

We write $u \models P$ whenever $u \models Z$. For $u \models P$ we define $\text{Cost}(u, P)$ as follows:

$$\text{Cost}(u, P) = c + \sum_{x \in C} r(x) \cdot (u(x) - \Delta_Z(x))$$

$$\text{Cost}(x, y) = 2y - x + 2$$



Branch & Bound Algorithm

Cost := ∞

Passed := \emptyset

Waiting := $\{(l_0, Z_0)\}$

while Waiting $\neq \emptyset$ **do**

select (l, Z) from Waiting

if $l = l_g$ and minCost(Z) < Cost **then**

 Cost := minCost(Z)

if minCost(Z) + Rem_(l, Z) \geq Cost **then** break

if for all (l, Z') in Passed: $Z' \not\leq Z$ **then**

add (l, Z) to Passed

add all (l', Z') with $(l, Z) \rightarrow (l', Z')$ to Waiting

return Cost

Branch & Bound Algorithm

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if $l = l_g$ and minCost(Z) < Cost **then**

 Cost := minCost(Z)

if minCost(Z) + Rem_(l, Z) \geq Cost **then**

if for all (l, Z') in Passed: $Z' \not\leq Z$ **then**

 add (l, Z) to Passed

 add all (l', Z') with $(l, Z) \rightarrow (l', Z')$

return Cost

$Z' \leq Z$

Z' is bigger &
cheaper than Z

\leq is a well-quasi
ordering which
guarantees
termination!

Experimental Results

Modeling APL with PDDL₃

[Dierks VVPS05]

```
(define (domain Planes)
  (:requirements :typing :fluents :negative-preconditions
    :conditional-effects :equality :duration-inequalities
    :durative-actions)
  (:types plane runway)
  (:predicates  (occupied ?r - runway)
    (landed    ?p - plane)
    (scheduled ?p - plane ?r - runway))
  (:functions   (earliest      ?p - plane)
    (target        ?p - plane)
    (latest        ?p - plane)
    (early-rate    ?p - plane)
    (late-rate     ?p - plane)
    (late-penalty  ?p - plane)
    (cost)))
```

```
(:durative-action land-late      :parameters (?p - plane ?r - runway)
:duration (<= ?duration (- (latest ?p) (earliest ?p)))
:condition (and               (at end   (occupied ?r))
              (at end   (scheduled ?p ?r))
              (at start (= total-time (target ?p))))
:effect (and (at end (landed ?p))
              (at start (increase cost (late-penalty ?p)))
              (increase cost (late-rate ?p))))
)
```

Modeling APL with PDDL₃

[Dierks VVPS05]

```
(define (problem plane1)
  (:domain Planes)
  (:objects      KL101,KL108,KL115 - plane
                 r - runway)

  (:init (not (occupied r))
         (not (landed   KL101)) (not (scheduled KL101 r))
         (not (landed   KL108)) (not (scheduled KL108 r))
         (not (landed   KL115)) (not (scheduled KL115 r))

         (= (earliest KL101) 129) (= (early-rate     KL101) 10)
         (= (target    KL101) 155) (= (late-rate      KL101) 10)
         (= (latest     KL101) 559) (= (late-penalty  KL101) 500)

         (= (earliest KL108) 195) (= (early-rate     KL108) 10)
         (= (target    KL108) 258) (= (late-rate      KL108) 10)
         (= (latest     KL108) 744) (= (late-penalty  KL108) 500)

         (= (earliest KL115) 89)  (= (early-rate     KL115) 30)
         (= (target    KL115) 98)  (= (late-rate      KL115) 30)
         (= (latest     KL115) 510) (= (late-penalty  KL115) 500)
  )

  (:goal (and   (landed KL101)
                (landed KL108)
                (landed KL115)
                (not (occupied r))))
         (:metric minimize cost)
  )
)
```

Experimental Results

Translating PDDL₃ to PTA

[Dierks VVPS05+]

# planes	# inst	# clocks	costs	time (s)	mem (MB)		
3	2	3	0	0.5	5		
3	3	4	0	1.9	9		
3	4	5	0	9	28		
4	2	3	860	0.8	7		
4	3	4	860	7.4	29		
4	4	5	860	59	168		
5	2	3	1540	2.4	16		
5	3	4	1540	30	114		
5	4	5	out of mem (400MB, 97 s)				
6	2	3	180	10.3	56		
6	3	4	60	out of mem (96 s)			
6	4	5	out of mem (76 s)				
7	2	3	920	30.8	166		
7	3	4	out of mem (66 s)				
8	2	3	1870	out of mem (63 s)			
8	3	4	out of mem (60 s)				
9	2	3	out of mem (55 s)				

Experimental Results

PTA versus MILP

[Beasley'00]

RW	Planes	10	15	20	20	20	30	44
	Types	2	2	2	2	2	4	2
1	MILP (s)	0.4	5.2	2.7	220.4	922.0	33.1	10.6
	MC (s)	0.8	5.6	2.8	20.9	49.9	0.6	2.2
	Factor	2.0	1.08	1.04	10.5	18.5	55.2	48.1
2	MILP (s)	0.6	1.8	3.8	1919.9	11510.4	1568.1	0.2
	MC (s)	2.7	9.6	3.9	138.5	187.1	6.0	0.9
	Factor	4.5	5.3	1.02	13.9	61.5	261.3	4.5
3	MILP (s)	0.1	0.1	0.2	2299.2	1655.3	0.2	N/A
	MC (s)	0.2	0.3	0.7	1765.6	1294.9	0.6	
	Factor	2.0	3.0	3.5	1.30	1.28	3.0	
4	MILP (s)	N/A	N/A	N/A	0.2	0.2	N/A	N/A
	MC (s)				3.3	0.7		
	Factor				16.5	3.5		

Experimental Results

OPTOP PDDL₃

To be provided with guidance
from
Maria Fox & Drew McDermott

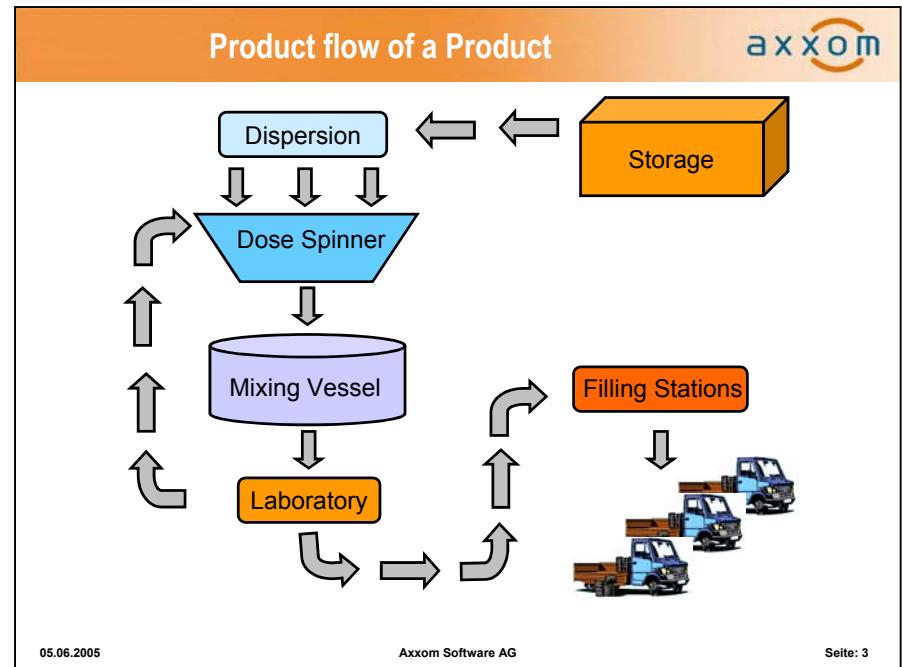
Overview

- Timed Automata & Scheduling
- Priced Timed Automata and Optimal Scheduling
- AMETIST Result
- Beyond Static Scheduling

AXXOM Case study

Laquer Production Scheduling

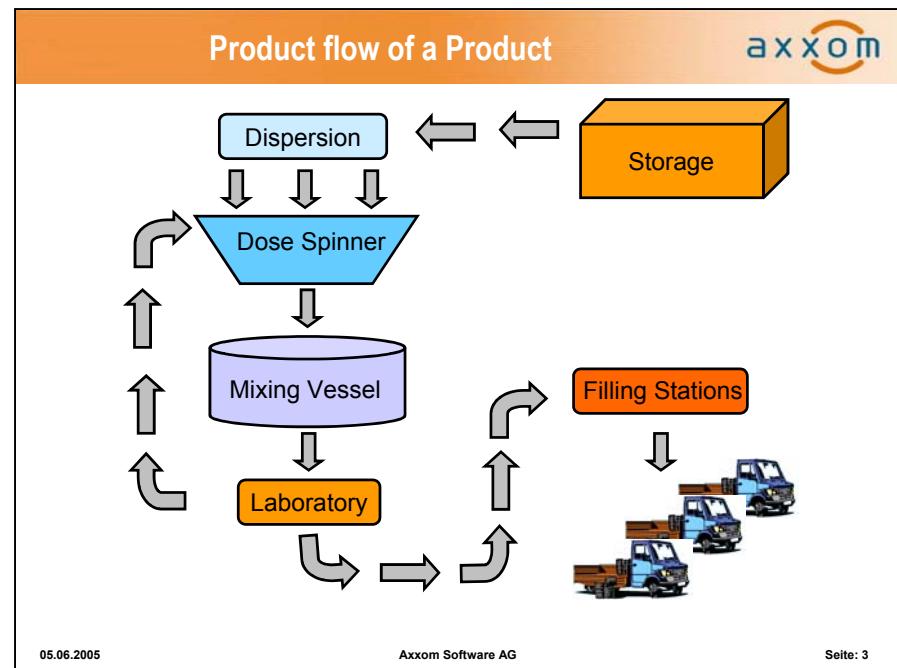
- 3 types of recipes
 - for uni/metallic/bronze
 - use of resources, processing times, timing
- 29 (73, 219) orders:
 - start time, due date, recipe
- extensions:
 - delay cost, storage cost, setup cost
 - weekend, nights



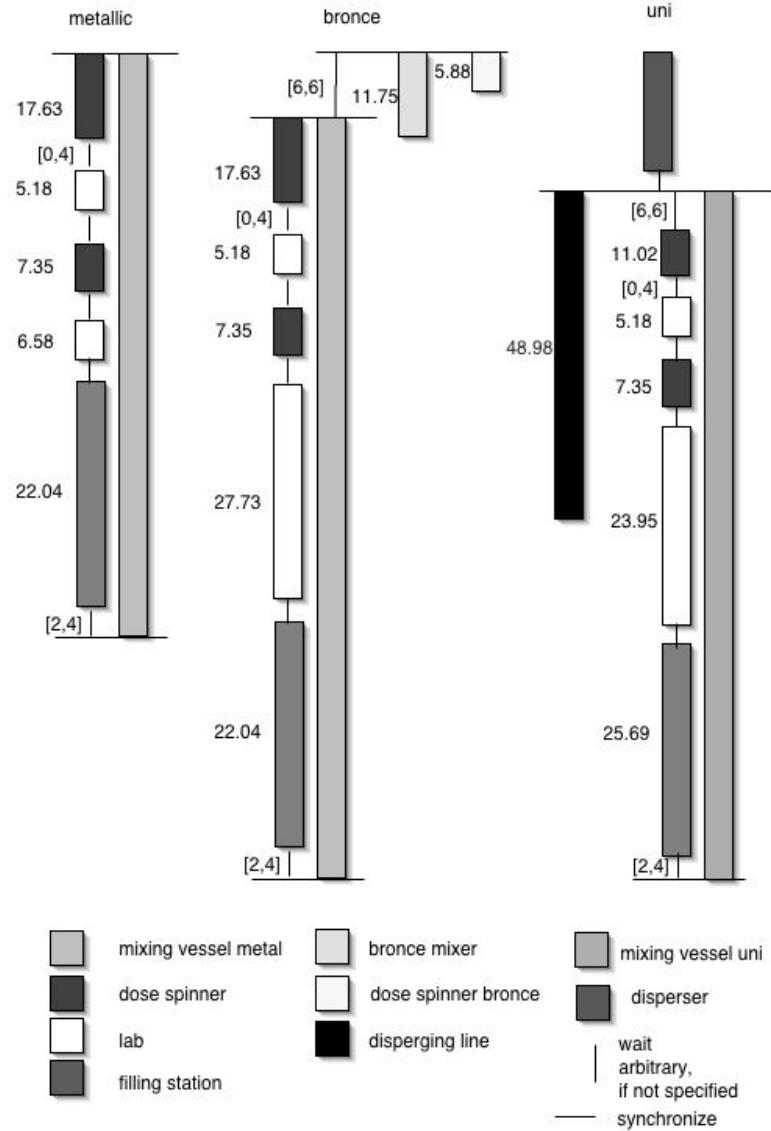
Behrmann, Brinksma, Hendriks, Mader
16th IFAC World Congress

Resources

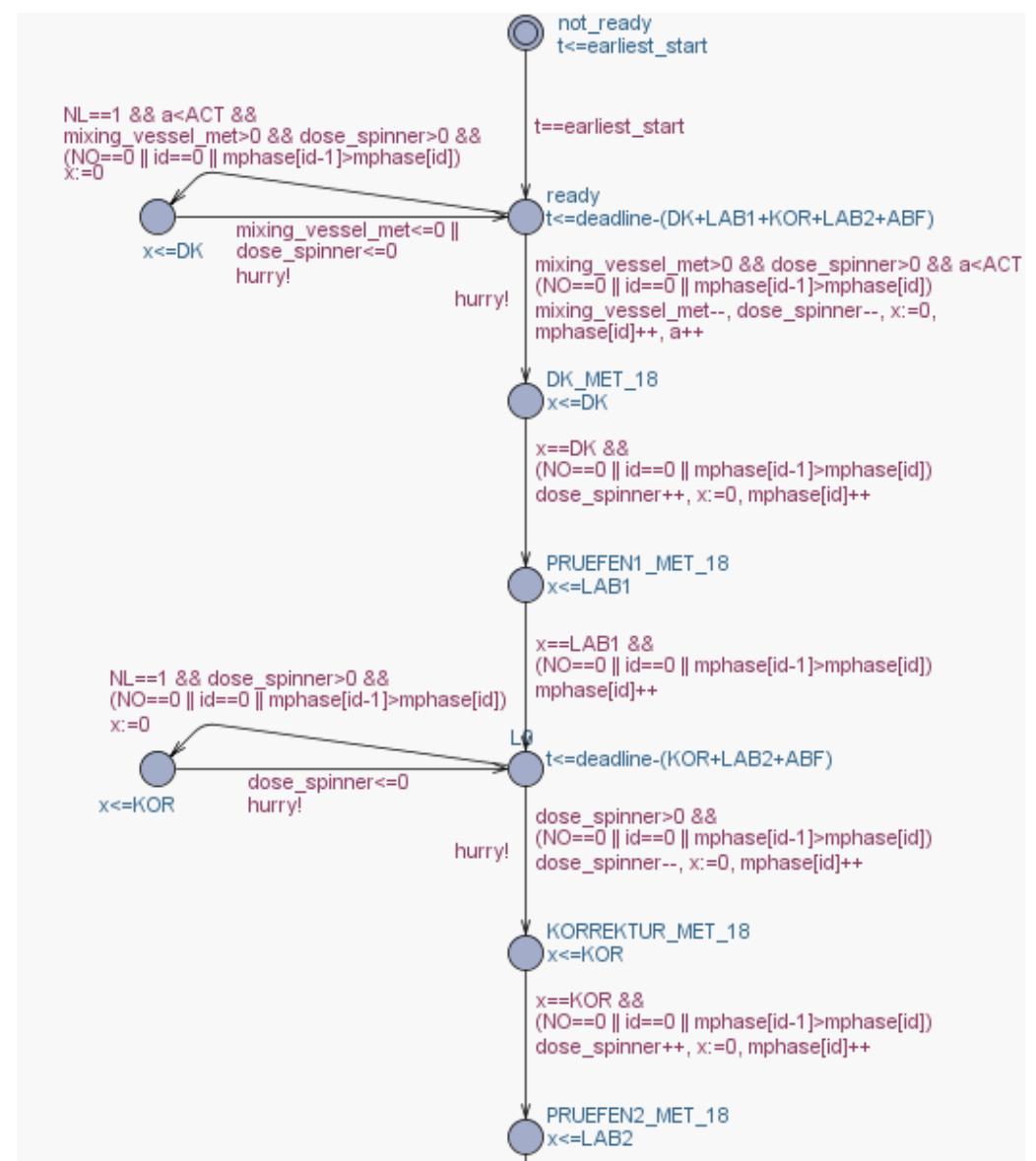
- 2 mixing vessels for uni lacquers
- 3 mixing vessels for metallic/bronze
- 2 dose spinners
- 1 dose spinner bronze
- 1 dispersing line
- 1 predisperser
- 1 bronze mixer
- 2 filling lines
- lab (unlimited)



Recipes



UPPAAL template for metal



Orders

order	product ID	quantity	unit	delay costs per day per kg	earliest start	delivery date
1204081.FE	10009.ABF.MET.18	19000	kg	51	05.04.2002 19:12	19.04.2002 19:12
4147108.FE	10009.ABF.MET.18	19000	kg	51	20.04.2002 06:00	04.05.2002 06:00
4152856.FE	10009.ABF.MET.18	19000	kg	51	29.03.2002 07:12	12.04.2002 07:12
4161304.FE	10001.ABF.UNI.85	23000	kg	51	08.04.2002 04:00	22.04.2002 04:00
4173559.FE	10001.ABF.UNI.85	22000	kg	0,51	01.04.2002 22:48	15.04.2002 22:48
4173830.FE	10001.ABF.UNI.85	22000	kg	0,51	15.04.2002 02:48	29.04.2002 02:48
4174631.FE	10009.ABF.MET.18	19000	kg	51	22.04.2002 22:48	06.05.2002 22:48
4179351.FE	10009.ABF.MET.18	19000	kg	51	01.04.2002 23:59	15.04.2002 23:59
4179360.FE	10009.ABF.MET.18	19000	kg	51	29.03.2002 19:12	12.04.2002 19:12
4187241.FE	10001.ABF.UNI.85	22000	kg	0,51	15.04.2002 22:48	29.04.2002 22:48
4187676.FE	10009.ABF.MET.18	19000	kg	51	12.04.2002 19:12	26.04.2002 19:12
4187688.FE	10009.ABF.MET.18	19000	kg	51	12.04.2002 23:59	26.04.2002 23:59
4189165.FE	10009.ABF.MET.18	19000	kg	51	01.04.2002 04:00	15.04.2002 04:00
4189694.FE	10001.ABF.UNI.85	22000	kg	51	03.04.2002 10:24	17.04.2002 10:24
4196932.FE	10001.ABF.UNI.85	22000	kg	51	25.03.2002 04:39	08.04.2002 04:39
4196968.FE	10001.ABF.UNI.85	22000	kg	51	02.05.2002 14:24	16.05.2002 14:24
4197088.FE	10009.ABF.MET.18	19000	kg	51	27.03.2002 04:48	10.04.2002 04:48
4197955.FE	10001.ABF.UNI.85	22000	kg	51	03.04.2002 14:24	17.04.2002 14:24
4200741.FE	10009.ABF.MET.18	19000	kg	51	16.04.2002 04:48	30.04.2002 04:48
4206483.FE	10037.ABF.MET.11	19000	kg	51	15.04.2002 23:59	29.04.2002 23:59
4206683.FE	10037.ABF.MET.11	19000	kg	51	15.04.2002 04:00	29.04.2002 04:00
4209227.FE	10009.ABF.MET.18	19000	kg	51	12.04.2002 14:24	26.04.2002 14:24
4210646.FE	10001.ABF.UNI.85	22000	kg	51	26.04.2002 14:24	10.05.2002 14:24
4210669.FE	10037.ABF.MET.11	19000	kg	51	08.04.2002 04:00	22.04.2002 04:00
4213014.FE	10009.ABF.MET.18	19000	kg	51	23.03.2002 02:24	06.04.2002 02:24
4213016.FE	10009.ABF.MET.18	19000	kg	51	20.04.2002 02:24	04.05.2002 02:24
4220296.FE	10009.ABF.MET.18	19000	kg	51	27.03.2002 09:36	10.04.2002 09:36
4222941.FE	10037.ABF.MET.11	19000	kg	0,51	03.04.2002 06:00	17.04.2002 06:00
4223024.FE	10037.ABF.MET.11	19000	kg	0,51	18.04.2002 06:00	02.05.2002 06:00

Instantiated Model



Heuristics

■ Nice heuristics

- non-overtaking
 - orders of the same recipe cannot overtake each other
- non-laziness
 - a process that needs an available resource will not waste time if its is not claimed by others (a.k.a. active scheduling)

■ Cut-and-Pray heuristics

- greediness
 - a process that needs an available resource will claim this resource immediately
- reducing active orders
 - the number of concurrent orders is restricted (number of critical resources can give an indication)

Experimental Results

#jobs	heuristic	max. orders	term. time
29	-	-	-
29	nl	-	1 s
73	nl, no	-	-
73	nl, no	3	7 s
219	g, no	4	8 s

uses clock optimization &
optimized successor
calculation

Results Extended Case

storage, delay and setup costs, working hours

#jobs	work hrs	is†	max
29	2,263,496	-	14,875
29	192,881,129	no	4
29	exp	o, g	-

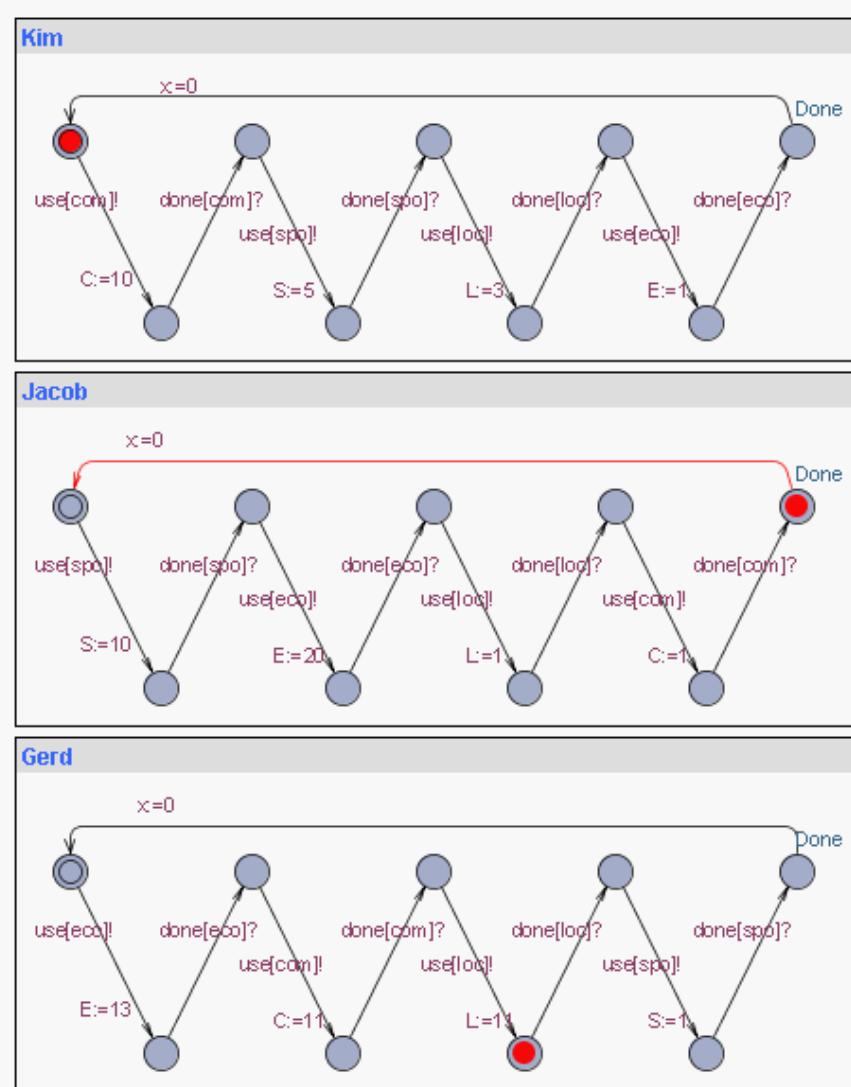
Order of magnitude faster than MILP, GAMS/CPLEX

Competitive with Orion-pi results

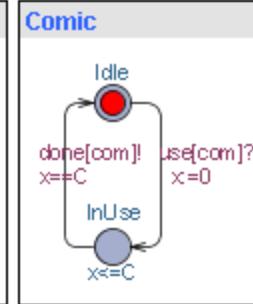
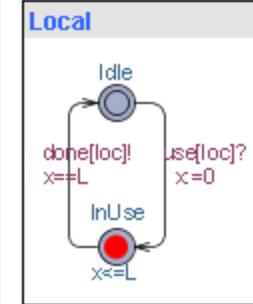
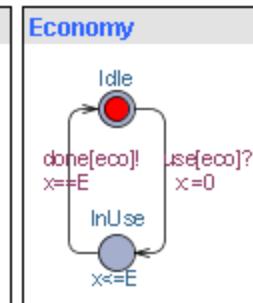
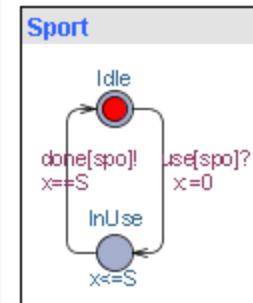
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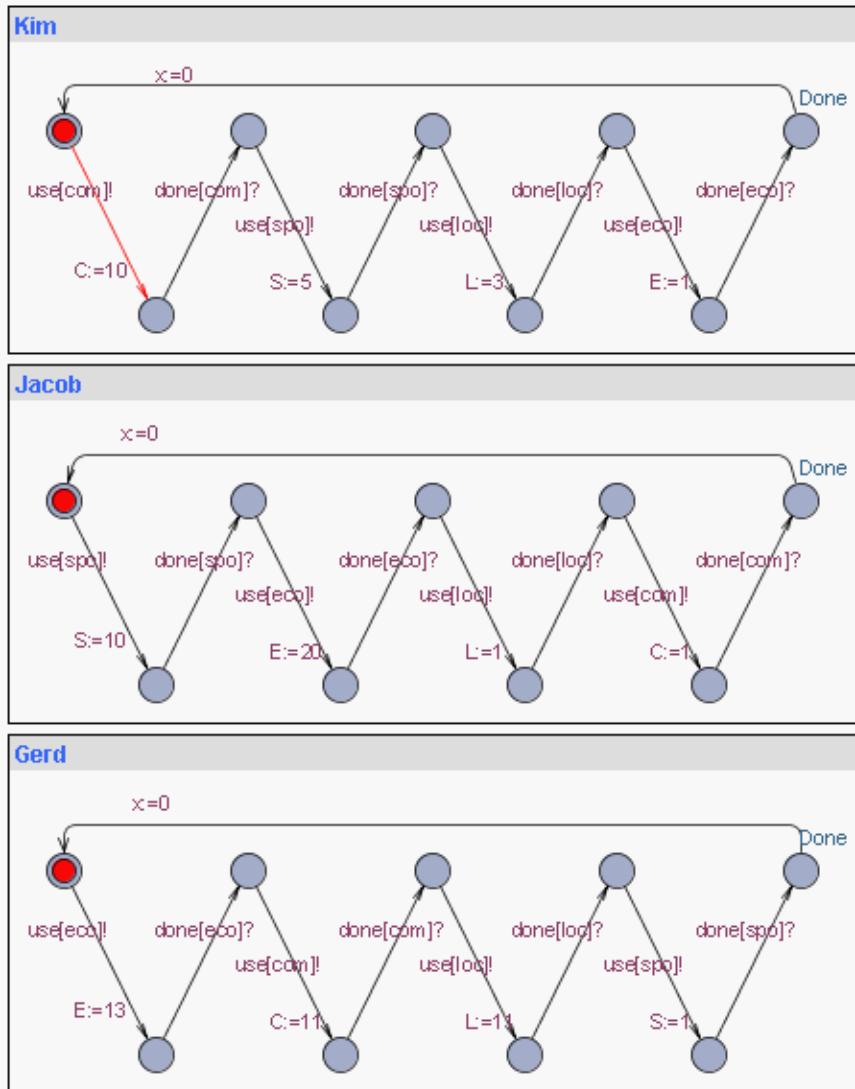
Infinite Scheduling



$E[] (\text{Kim}.x \leq 100 \text{ and } \text{Jacob}.x \leq 90 \text{ and } \text{Gerd}.x \leq 100)$

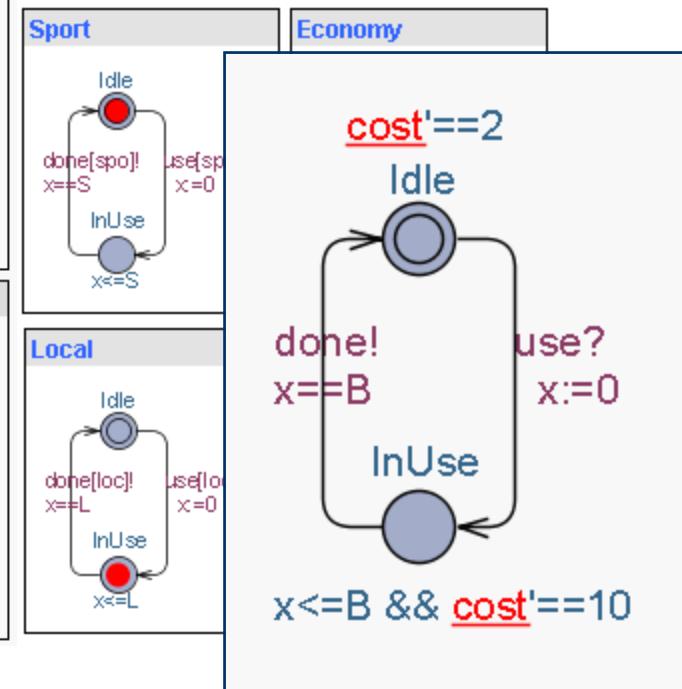


Infinite Optimal Scheduling

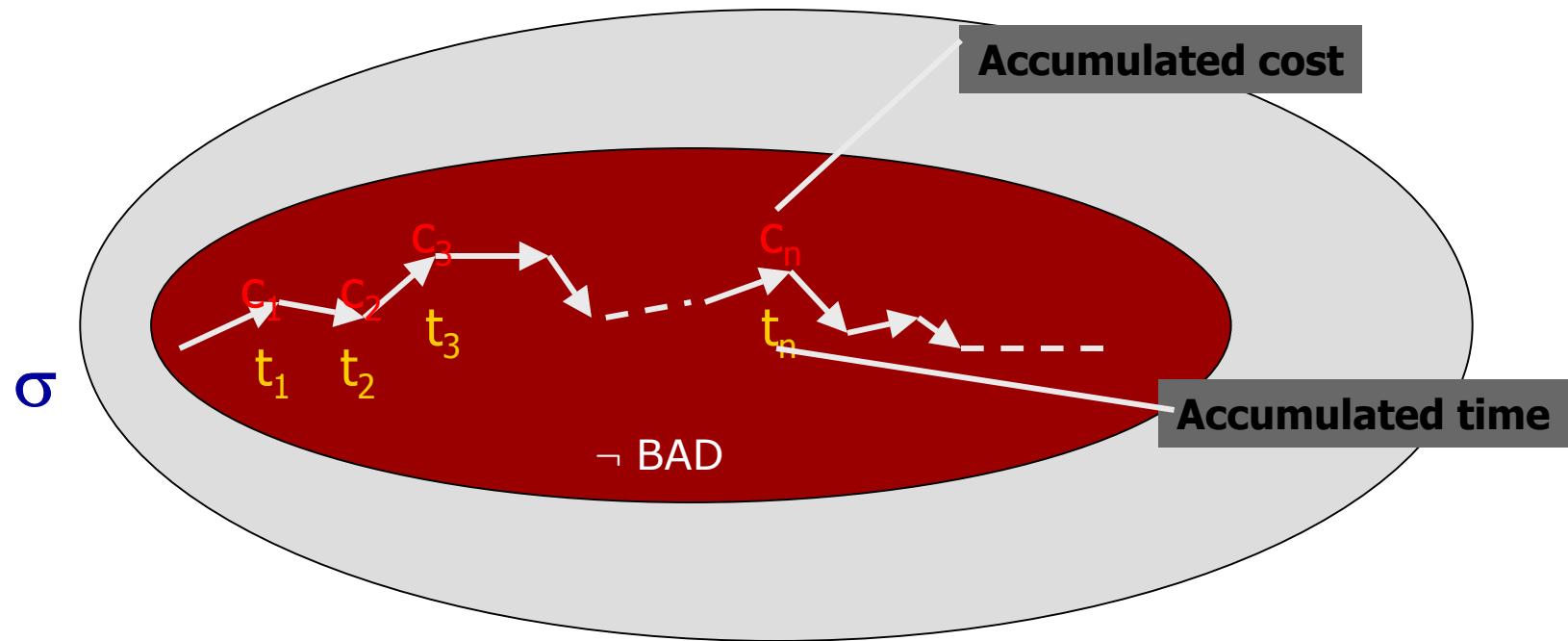


$E[] \text{ (Kim.x } \leq 100 \text{ and }$
 $\text{Jacob.x } \leq 90 \text{ and }$
 $\text{Gerd.x } \leq 100 \text{)}$

+ most minimal limit
of cost/time



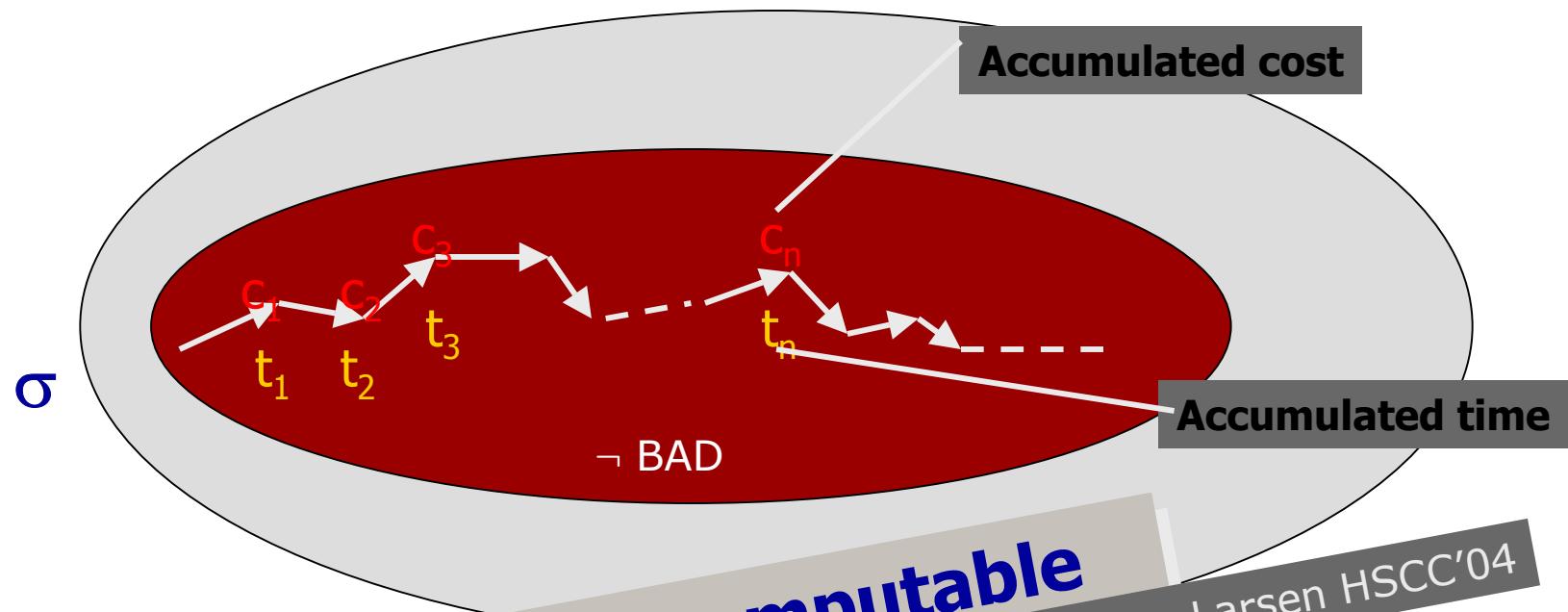
Infinite Optimal Scheduling



Value of path σ : $\text{val}(\sigma) = \lim_{n \rightarrow \infty} c_n/t_n$

Optimal Schedule σ^* : $\text{val}(\sigma^*) = \inf_{\sigma} \text{val}(\sigma)$

Infinite Optimal Scheduling



THEOREM: σ^* is computable

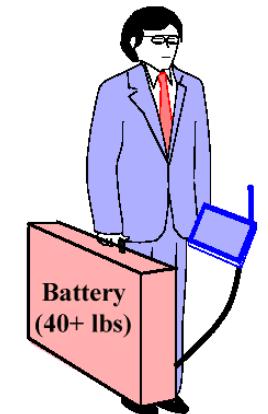
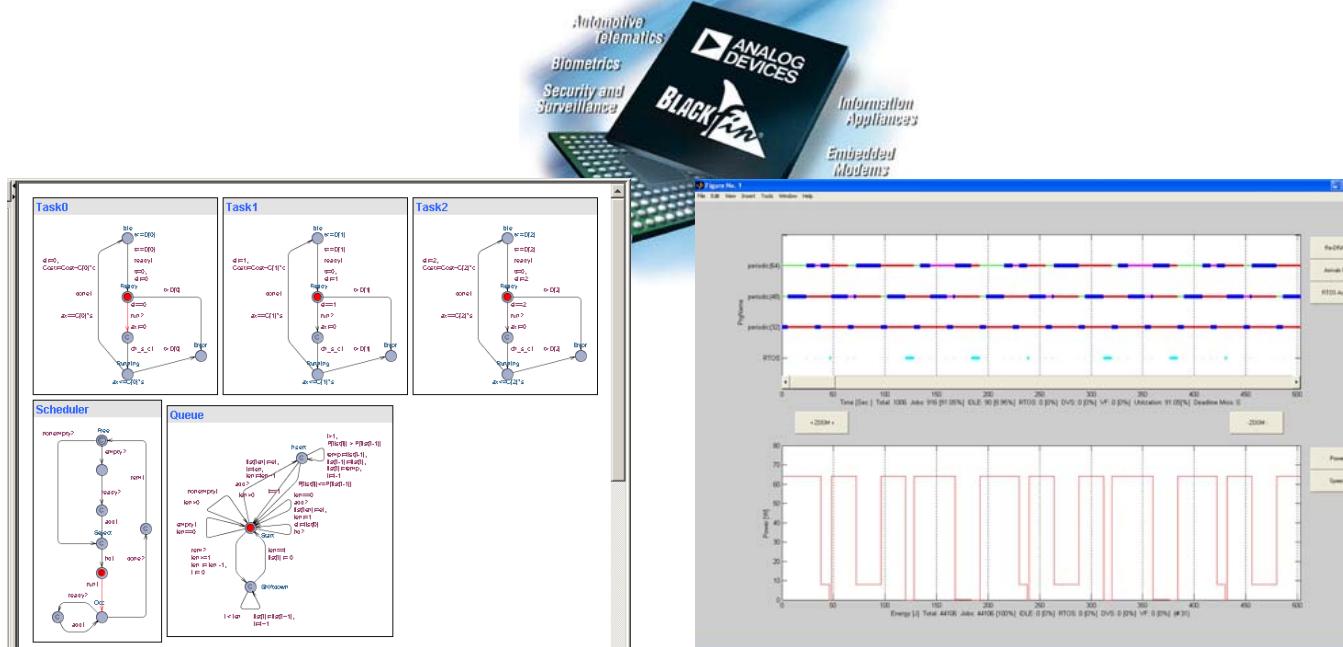
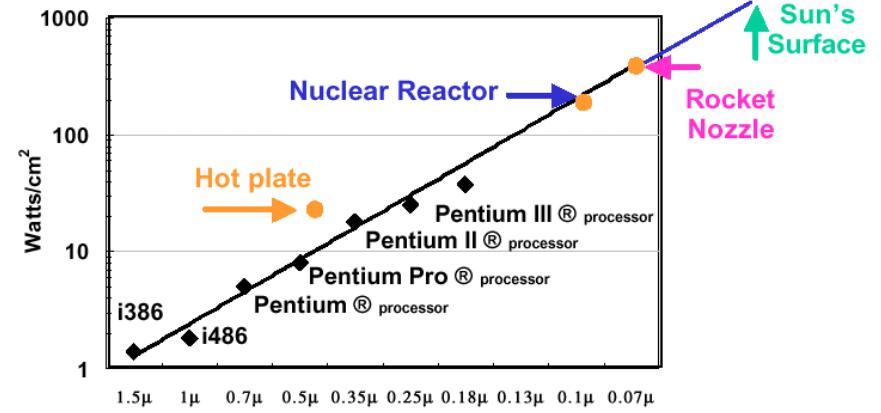
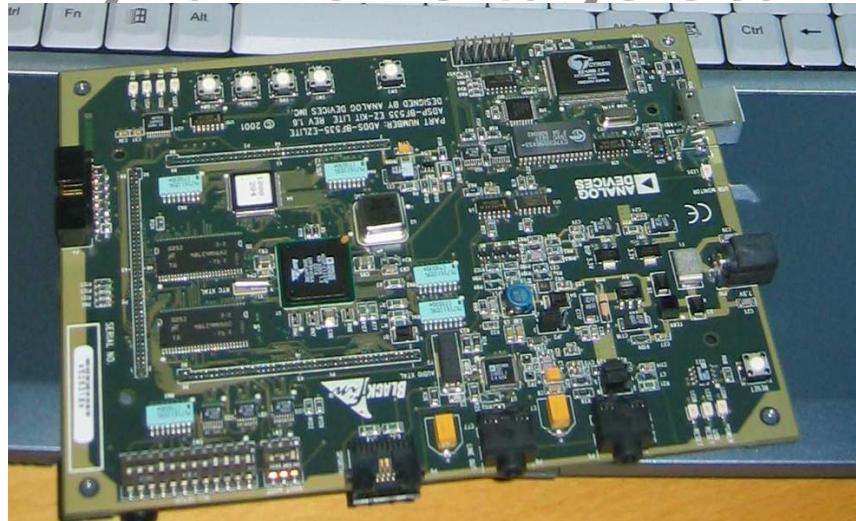
Bouyer, Brinksma, Larsen HSCC'04

$$\text{For all } \sigma: \text{val}(\sigma) = \lim_{n \rightarrow \infty} c_n/t_n$$

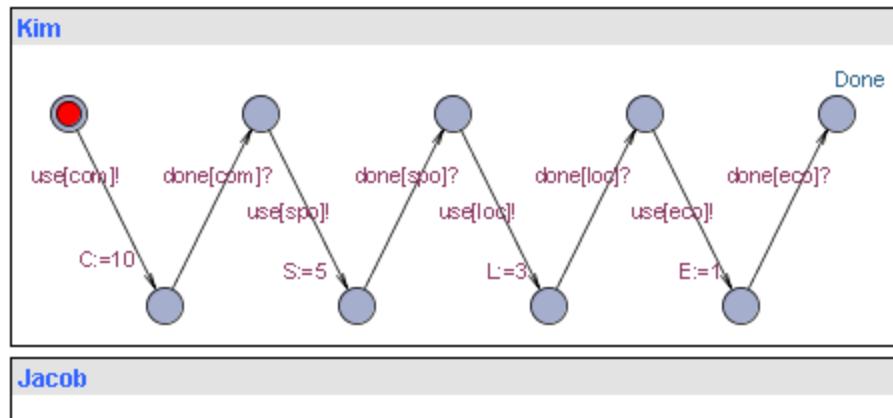
Optimal Schedule σ^* : $\text{val}(\sigma^*) = \inf_{\sigma} \text{val}(\sigma)$

Application

Dynamic Voltage Scaling

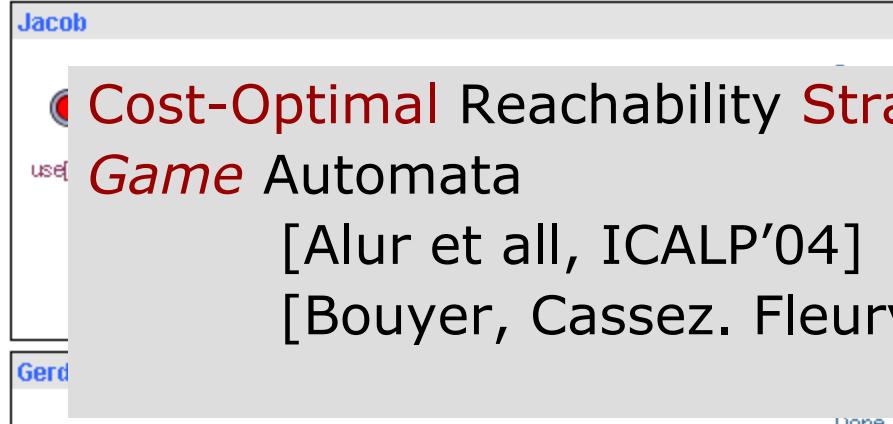


Dynamic Scheduling



$E <>$ (Kim.Done and
Jacob.Done and
Gerd.Done)

+ winning / optimal
strategy



Uncontrollable
timing uncertainty

Future Work & Conclusion

- PDDL₃ versus PTA
- OPTOP versus UPPAAL Cora
- Planning versus Model Checking

- Improved heuristic search
 - FF's relaxed planning approach
 - pattern databases
 - beam search
 - externalization
 - ...

- Please visit www.uppaal.com

Thank you for your attention !

